

# Software implementation and testing of GARCH models

G F LEVY

NAG Ltd, Wilkinson House,  
Jordan Hill Road,  
Oxford, OX2 8DR, U.K.  
(email: [george@nag.co.uk](mailto:george@nag.co.uk))

**Abstract:** This paper describes the software implementation of GARCH routines in Fortran 77. The routines considered here cover both symmetric and asymmetric GARCH and also shocks having Gaussian and non-Gaussian distributions. Extensive examples of using the software are provided and the test results from Monte Carlo simulations are presented.

**Keywords:** GARCH, ARCH, maximum likelihood estimation, volatility, generalised autoregressive heteroskedasticity, asymmetry, Fortran 77

## 1 Introduction

The modelling of sequences with time-dependent variance is crucial to many areas of mathematical finance. This technical report describes the software implementation and testing of a set of univariate generalised autoregressive conditional heteroskedastic (GARCH) routines that have been developed at NAG Ltd for its numerical libraries. Although we are primarily concerned with GARCH software that has been developed for the next release of the NAG Fortran 77 Library [1], some GARCH software is already contained in the current version of the NAG C Library [2]. The Fortran 77 software described here can be used for GARCH( $p,q$ ) models with arbitrary values of  $p$  and  $q$ . There are routines for the generation of GARCH sequences, model estimation and volatility forecasting. The estimation routines return not only the parameter estimates but also other important statistics such as: the standard errors, the scores and the value of the log-likelihood function for the calculated model parameters.

Other capabilities of the software include the following:

- regression-GARCH( $p,q$ ) models
- symmetric models
- asymmetric models
- shocks with Gaussian and non-Gaussian distributions

Since both the NAG Fortran 77 and C libraries are implemented as Dynamic Link Libraries (DLLs) the GARCH software can be easily used from the Microsoft Windows environment. This means that the GARCH routines can be readily incorporated into Microsoft software such as Excel, Visual Basic, etc.

## 2 The GARCH models

The standard (symmetric) regression-GARCH( $p,q$ ) model [3] [4] [5], with Gaussian shocks  $\mathbf{e}_t$ , takes the following form:

$$y_t = b_0 + x_t^T \mathbf{b} + \mathbf{e}_t, \quad \mathbf{e}_t | \mathbf{y}_{t-1} = N(0, h_t)$$

$$h_t = \mathbf{a}_0 + \sum_{i=1}^q \mathbf{a}_i \mathbf{e}_{t-i}^2 + \sum_{j=1}^p \mathbf{b}_j h_{t-j}$$

This process is described by  $q + 1$  coefficients  $\mathbf{a}_i$ ,  $i = 0, \dots, q$ ,  $p$  coefficients  $\mathbf{b}_i$ ,  $i = 1, \dots, p$ , mean  $b_0$ ,  $k$  linear regression coefficients  $b_i$ ,  $i = 1, \dots, k$ , endogenous/exogenous variables  $y_t$  and  $x_t$  respectively, shocks  $\mathbf{e}_t$ , conditional variance  $h_t$ , and the set of all information up to time  $t$ ,  $\mathbf{y}_t$ .

It should be noted that for  $p = 0$  a GARCH( $p,q$ ) model is also called an ARCH( $q$ ) model.

Empirical studies on financial time series have shown that they are characterised by increased conditional variance  $h_t$  following negative shocks (bad news). The distribution of the shocks have been also found to exhibit considerable leptokurtosis. Since the standard Gaussian GARCH model cannot capture these effects various GARCH model extensions have been developed [6].

The asymmetric GARCH models considered here are:

AGARCH( $p,q$ )-type1

$$h_t = \mathbf{a}_0 + \sum_{i=1}^q \mathbf{a}_i (\mathbf{e}_{t-i} + \mathbf{g})^2 + \sum_{j=1}^p \mathbf{b}_j h_{t-j}$$

AGARCH( $p,q$ )-type2

$$h_t = \mathbf{a}_0 + \sum_{i=1}^q \mathbf{a}_i (|\mathbf{e}_{t-i}| + \mathbf{g}\mathbf{e}_{t-i})^2 + \sum_{j=1}^p \mathbf{b}_j h_{t-j}$$

GJR-GARCH( $p,q$ ), or Glosten, Jagannathan and Runkle GARCH [7]

$$h_t = \mathbf{a}_0 + \sum_{i=1}^q (\mathbf{a}_i + \mathbf{g}S_{t-i}) \mathbf{e}_{t-i}^2 + \sum_{j=1}^p \mathbf{b}_j h_{t-j}$$

where  $S_t = 1$  if  $\mathbf{e}_t < 0$  and  $S_t = 0$  if  $\mathbf{e}_t \geq 0$

EGARCH( $p,q$ ), or exponential GARCH

$$\ln(h_t) = \mathbf{a}_0 + \sum_{i=1}^q \mathbf{a}_i z_{t-i} + \sum_{i=1}^q \mathbf{f}_i (|z_{t-i}| - E[|z_{t-i}|]) + \sum_{j=1}^p \mathbf{b}_j \ln(h_{t-j})$$

where  $z_t = \frac{\mathbf{e}_t}{\sqrt{h_t}}$  and  $E[|z_{t-i}|]$  denotes the expected value of  $|z_{t-i}|$

In all these models the shocks,  $\mathbf{e}_t$ , can either have a Gaussian distribution or Student's  $t$ -distribution with a specified number of degrees of freedom.

In AGARCH-type1 the asymmetric effects are modelled via the extra parameter  $\mathbf{g}$ . For example, in the standard GARCH(1,1) model when  $h_{t-1}$  is fixed  $h_t = h(\mathbf{e}_{t-1})$  is a parabola with a minimum at  $\mathbf{e}_{t-1} = 0$ . The introduction of the additional parameter  $\mathbf{g}$  shifts the parabola horizontally so that the minimum occurs at  $\mathbf{e}_{t-1} = -\mathbf{g}$ . The conditional variance following negative shocks can therefore be enhanced by choosing  $\mathbf{g} < 0$ , so that  $h(-\mathbf{e}_{t-1}) > h(\mathbf{e}_{t-1})$  for  $\mathbf{e}_{t-1} > 0$ .

In an AGARCH-type2 model the inclusion of  $\mathbf{g}$  can also result in an enhancement of  $h_t$  following a negative shock  $\mathbf{e}_{t-1}$ . For a GARCH(1,1) model  $h(-\mathbf{e}_{t-1}) > h(\mathbf{e}_{t-1})$  for  $\mathbf{e}_{t-1} > 0$  and  $\mathbf{g} < 0$ .

Similarly in the GJR-GARCH(1,1) model the value of  $h_t$  is increased above the symmetric case when  $\mathbf{e}_{t-1} < 0$  and  $\mathbf{g} > 0$ .

For EGARCH, asymmetric response arises from the terms  $\sum_{i=1}^q \mathbf{a}_i z_{t-i}$ . In an EGARCH(1,1), if  $\mathbf{a}_1 < 0$  then a negative shock  $\mathbf{e}_{t-1} < 0$  increases the value of  $h_t$ , that is  $\ln\{h(-z_{t-1})\} > \ln\{h(z_{t-1})\}$ .

All the GARCH processes above are uniquely described by the parameter vector  $\mathbf{q}$ , where  $\mathbf{q} = (b_0, b^T, \mathbf{w}^T)$ ,  $\mathbf{w}^T = (\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_q, \mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_p, \mathbf{g})$  and  $b^T = (b_1, \dots, b_k)$ .

The GARCH model implementations here all rely on finding the value of  $\mathbf{q}$  which maximises the conditional log-likelihood (objective) function

$$lf = \frac{1}{2} \sum_{i=1}^T \left( \log(h_i) + \frac{\mathbf{e}_i^2}{h_i} \right)$$

where  $T$  is the number of terms in the sequence.

This is achieved by starting with an initial approximation for  $\mathbf{q}$  and then using numerical optimisation to iterate to an acceptable solution. In all the GARCH estimation routines, apart from EGARCH, an analytic expression for the Jacobian of the objective function was used for the optimisation stage.

The standard errors for the parameter estimates can be computed by using the well known result [8] that the maximum likelihood estimate for  $\mathbf{q}$  is asymptotically normal with mean  $\mathbf{q}$  and covariance matrix  $\mathfrak{S}^{-1}$  where  $\mathfrak{S}$  (The Fisher Information Matrix) is given by:

$$\mathfrak{S} = \mathbb{E} \left[ \sum_{i=1}^T \frac{\partial^2 lf}{\partial \mathbf{q} \partial \mathbf{q}^T} \right]$$

## 3 The GARCH software

Here we list the available software for sequence generation, model parameter estimation, and forecasting of regression-GARCH( $p, q$ ) sequences.

### 3.1 Generation

The following routines generate a given number of terms from various symmetric and asymmetric GARCH( $p, q$ ) sequences.

#### Specification

##### *AGARCH-type1*

*SUBROUTINE G05HKF (DIST, NUM, IP, IQ, THETA, GAMMA, DF, HT, YT, FCALL, RVEC, IFLAG)*

## AGARCH-type2

SUBROUTINE G05HLF (DIST, NUM, IP, IQ, THETA, GAMMA, DF, HT, YT, FCALL, RVEC, IFLAG)

## GJR-GARCH

SUBROUTINE G05HMF (DIST, NUM, IP, IQ, THETA, GAMMA, DF, HT, YT, FCALL, RVEC, IFLAG)

## EGARCH

SUBROUTINE G05HNF (DIST, NUM, IP, IQ, THETA, DF, HT, YT, FCALL, RVEC, IFLAG)

The routine parameters have the following meanings:

### Parameters

*DIST* - CHARACTER\*1.

On entry, the type of distribution to use for  $e_i$   
if *DIST* = 'N' then a Normal distribution is used.  
if *DIST* = 'T' then a Student's *t*-distribution is used.

*NUM* - INTEGER

On entry, the number of terms in the sequence, *T*

*IP* - INTEGER.

On entry, the number of moving average coefficients, *p*

*IQ* - INTEGER.

On entry, the number of auto-regressive coefficients, *q*

*THETA* - DOUBLE PRECISION array.

On entry, the parameters of the GARCH model

*GAMMA* - DOUBLE PRECISION.

On entry, the asymmetry parameter  $g$  for the GARCH sequence.

*DF* - DOUBLE PRECISION

On entry, the number of degrees of freedom for the Student's *t*-distribution.  
It is not referenced if *DIST* = 'N'

*HT* - DOUBLE PRECISION array.

On exit, the conditional variances for the GARCH sequence,  $h_i, i = 1, \dots, T$

*YT* - DOUBLE PRECISION array.

On exit, the observations for the GARCH sequence,  $e_i, i = 1, \dots, T$ .

*FCALL* - LOGICAL.

On entry, if *FCALL* = *.TRUE.* then a new sequence is to be generated,  
else a given sequence is to be continued using the information in *RVEC*.

*RVEC* - DOUBLE PRECISION array.

On entry, the array contains information required to continue a sequence if *FCALL* = *.FALSE.*  
On exit, contains information that can be used in a subsequent call, with *FCALL* = *.FALSE.*

*IFLAG* - INTEGER.

The error indicator

## 3.2 Estimation

The following routines estimate the model parameters of various symmetric and asymmetric regression-GARCH( $p,q$ ) sequences.

### Specification

#### AGARCH-type1

SUBROUTINE G13FAF (DIST, YT, X, LDX, NUM, IP, IQ, NREG, MN, ISYM, THETA, SE, SC, COVAR, LDC, HP, ETM, HTM, LGF, COPTS, MAXIT, TOL, WORK, LWORK, IFLAG)

#### AGARCH-type2

SUBROUTINE G13FCF (DIST, YT, X, LDX, NUM, IP, IQ, NREG, MN, THETA, SE, SC, COVAR, LDC, HP, ETM, HTM, LGF, COPTS, MAXIT, TOL, WORK, LWORK, IFLAG)

#### GJR-GARCH

SUBROUTINE G13FEF (DIST, YT, X, LDX, NUM, IP, IQ, NREG, MN, THETA, SE, SC, COVAR, LDC, HP, ETM, HTM, LGF, COPTS, MAXIT, TOL, WORK, LWORK, IFLAG)

#### EGARCH

SUBROUTINE G13FGF (DIST, YT, X, LDX, NUM, IP, IQ, NREG, MN, THETA, SE, SC, COVAR, LDC, HP, ETM, HTM, LGF, COPT, MAXIT, TOL, WORK, LWORK, IFLAG)

The routine parameters have the following meanings:

### Parameters

*DIST* - CHARACTER\*1

On entry, the type of distribution to use for  $e_j$ . If *DIST* = 'N' then a Normal distribution is used, if *DIST* = 'T' then a Student's t- distribution is used.

*YT* - DOUBLE PRECISION array.

On entry, the sequence of observations,  $e_j, j = 1, \dots, T$ .

*X* - DOUBLE PRECISION array.

On entry, the *i*th row of *X* contains the time dependent exogenous vector,  $x_i, i = 1, \dots, T$ .

*LDX* - INTEGER.

On entry, the first dimension of the array *X*

*NUM* - INTEGER.

On entry, the number of terms in the sequence, *T*

*IP* - INTEGER.

On entry, the number of moving average coefficients, *p*

*IQ* - INTEGER.

On entry, the number of auto-regressive coefficients, *q*

*NREG* - INTEGER.

On entry, the number of regression coefficients, *k*

*MN* - INTEGER.

On entry, if *MN* = 1 then the mean term  $b_0$  will be included in the model

*ISYM* - INTEGER.

On entry, if *ISYM* = 1 then the asymmetry term  $g$  will be included in the model (this only applies to G13FAF)

*THETA* - DOUBLE PRECISION array.

*On entry*, the initial parameter estimates for the vector  $\mathbf{q}$

*On exit*, the estimated values  $\hat{\mathbf{q}}$  for the vector  $\mathbf{q}$

*SE* - DOUBLE PRECISION array.

*On exit*, the standard errors for  $\hat{\mathbf{q}}$

*SC* - DOUBLE PRECISION array.

*On exit*, the scores for  $\hat{\mathbf{q}}$

*COVAR* - DOUBLE PRECISION array.

*On exit*, the covariance matrix of the parameter estimates  $\hat{\mathbf{q}}$ , that is the inverse of the Fisher Information Matrix.

*LDC* - INTEGER.

*On entry*, the first dimension of the array *COVAR*

*HP* - DOUBLE PRECISION

*On entry*, if *COPTS(2) = .FALSE.* then *HP* is the value to be used for the pre-observed of the conditional variance.

If *COPTS(1) = .TRUE.* then *HP* is not referenced.

*On exit*, if *COPTS(2) = .TRUE.* then *HP* is the estimated value of the pre-observed of the conditional variance.

*ET* - DOUBLE PRECISION array.

*On exit*, the estimated residuals,  $e_i, i = 1, \dots, T$ .

*HT* - DOUBLE PRECISION array.

*On exit*, the estimated conditional variances,  $h_i, i = 1, \dots, T$

*LGF* - DOUBLE PRECISION.

*On exit*, the value of the likelihood function at  $\hat{\mathbf{q}}$

*COPTS* - LOGICAL array

If *COPTS(1) = .TRUE.* then stationary conditions are enforced, otherwise they are not.

If *COPTS(2) = .TRUE.* then the routine provides initial parameter estimates of the regression terms, otherwise these are provided by the user.

*MAXIT* - INTEGER.

*On entry*, the maximum number of iterations to be used by the optimisation routine when estimating the GARCH parameters.

*TOL* - DOUBLE PRECISION

*On entry*, the tolerance to be used by the optimisation routine when estimating the GARCH parameters.

*WORK* - DOUBLE PRECISION array, workspace.

*LWORK* - INTEGER.

*On entry*, the size of the work array *WORK*.

*IFLAG* - INTEGER.

The error indicator.

### 3.3 Forecasting

The following routines compute the volatility forecast for various symmetric and asymmetric GARCH( $p, q$ ) sequences.

#### Specification

##### *AGARCH-type1*

*SUBROUTINE G13FBF* (*NUM, NT, IP, IQ, THETA, GAMMA, CVAR, HT, ET, IFLAG*)

##### *AGARCH-type2*

*SUBROUTINE G13FDF* (*NUM, NT, IP, IQ, THETA, GAMMA, CVAR, HT, ET, IFLAG*)

##### *GJR-GARCH*

*SUBROUTINE G13FFF* (*NUM, NT, IP, IQ, THETA, GAMMA, CVAR, HT, ET, IFLAG*)

##### *EGARCH*

*SUBROUTINE G13FHF* (*NUM, NT, IP, IQ, THETA, CVAR, HT, ET, IFLAG*)

The routine parameters have the following meanings:

#### Parameters

*NUM* - INTEGER.

On entry, the number of terms in the arrays *HT* and *ET* from the modelled sequence.

*NT* - INTEGER.

On entry, the forecast horizon, *C*

*IP* - INTEGER.

On entry, the number of moving average coefficients, *p*

*IQ* - INTEGER.

On entry, the number of auto-regressive coefficients, *q*

*THETA* - DOUBLE PRECISION array.

On entry, model parameters of the GARCH sequence.

*GAMMA* - DOUBLE PRECISION.

On entry, the asymmetry parameter *g* or the GARCH sequence.

*CVAR* - DOUBLE PRECISION array.

On exit, the forecast expected values of the conditional variance,

*HT* - DOUBLE PRECISION array.

On entry, the sequence of past conditional variances for the GARCH(*p, q*)

process,  $h_i, i = 1, \dots, T$ .

*ET* - DOUBLE PRECISION array.

On entry, the sequence of past residuals for the GARCH(*p, q*) process,  $e_i, i = 1, \dots, T$ .

*IFLAG* - INTEGER.

The error indicator.



## 4 Examples of usage

In this section we provide complete Fortran 77 source code to illustrate how the GARCH routines can be used in practice. Each example generates a given type of GARCH sequence models this using the appropriate GARCH estimation and then computes a volatility forecast. All the examples consider GARCH sequences having both Gaussian and Student's  $t$ -distribution shocks.

### 4.1 AGARCH-type1

In this example the following two models are considered:

- An AARCH(3)-type1 model with shocks from a Gaussian distribution and observations,  
 $y_t = b_0 + e_t$
- An AGARCH(1,2) -type2 model with shocks from a Student's  $t$ -distribution and observations,  $y_t = b_0 + x_t^1 b_1 + x_t^2 b_2 + e_t$

Sequences of 1500 observations are generated for both of these processes, and then modelled using initial parameter estimates of half the true values. The final model parameter estimates are then output and a four step ahead volatility forecast is calculated.

#### Fortran source code

```
INTEGER NPARMX,NUM
DOUBLE PRECISION ZERO
PARAMETER (NPARMX=10,NUM=1500,ZERO=0.0D0)
INTEGER NUM1,NREGMX,MXNT,NT
PARAMETER (NUM1=3000,NREGMX=10,MXNT=400)
DOUBLE PRECISION FAC1,GAMMA,HP,LGF,MEAN,TOL,XTERM
INTEGER I,IFLAG,IP,IQ,ISYM,K,LDX,LWK,MAXIT,MN,NPAR,NREG,SEED
LOGICAL FCALL
DOUBLE PRECISION BX(10),COVAR(NPARMX,NPARMX),ETM(NUM1),
+ HT(NUM1+10),HTM(NUM1),PARAM(NPARMX),
+ RVEC(40),SC(NPARMX),SE(NPARMX),THETA(NPARMX),
+ WK(NUM1*3+NPARMX+NREGMX*NUM1+20*20+1),X(NUM1,10),
+ YT(NUM1+10),CVAR(100)
LOGICAL COPTS(2)
CHARACTER*1 DIST
DOUBLE PRECISION DF
EXTERNAL E04UEF,G05HKF,G13FAF,G13FBF,G05CBF
INTRINSIC ABS,DBLE,SIN
WRITE(*,*)'G13FAF Example Program Results'
SEED = 111
NREG = 0
LDX = NUM1
BX(1) = 1.5D0
BX(2) = 2.5D0
BX(3) = 3.0D0
MEAN = 3.0D0
DO 5 I = 1,NUM
  FAC1 = DBLE(I)*0.01D0
  X(I,1) = 0.01D0 + 0.7D0*SIN(FAC1)
  X(I,2) = 0.5D0 + FAC1*0.1D0
  X(I,3) = 1.0D0
5 CONTINUE
ISYM = 1
MN = 1
GAMMA = -0.4D0
IP = 0
IQ = 3
PARAM(1) = 0.8D0
PARAM(2) = 0.6D0
PARAM(3) = 0.2D0
```

```

PARAM(4) = 0.1D0
NPAR = 1 + IQ + IP

LWK = NREG*NUM+3*NUM+NPAR+ISYM+MN+NREG+403
FCALL = .TRUE.
IFLAG = 0
DIST = 'N'
CALL G05CBF(SEED)
CALL G05HKF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,
+          FCALL,RVEC,IFLAG)
FCALL = .FALSE.
CALL G05HKF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,
+          FCALL,RVEC,IFLAG)
IFLAG = -1
DO 10 I = 1,NUM
  XTERM = ZERO
  DO 15 K = 1,NREG
    XTERM = XTERM + X(I,K)*BX(K)
15  CONTINUE
  IF (MN.EQ.1) THEN
    YT(I) = MEAN + XTERM + YT(I)
  ELSE
    YT(I) = XTERM + YT(I)
  END IF
10 CONTINUE
CALL E04UEF('Nolist')
CALL E04UEF('Print Level = 0')
COPTS(1) = .TRUE.
COPTS(2) = .TRUE.
MAXIT = 200
TOL = 1.0D-16
DO 12 I = 1,NPAR
  THETA(I) = PARAM(I)*0.5D0
12 CONTINUE
IF (ISYM.EQ.1) THEN
  THETA(NPAR+ISYM) = GAMMA*0.5D0
END IF
IFLAG = 0
CALL G13FAF(DIST,YT,X,LDX,NUM,IP,IQ,NREG,MN,ISYM,
+          THETA,SE,SC,COVAR,
+          NPARMX,HP,ETM,HTM,LGF,COPTS,MAXIT,TOL,WK,
+          LWK,IFLAG)
WRITE(*,*)
WRITE(*,*)'Gaussian distribution'
WRITE(*,*)
WRITE(*,*)
+ 'Parameter estimates Standard errors Correct values'

DO 33 I = 1, NPAR
  WRITE(*, '(F16.4,F18.4,F13.4)') THETA(I),
+          SE(I),PARAM(I)
33 CONTINUE
IF (ISYM.EQ.1) THEN
  WRITE(*, '(F16.4,F18.4,F13.4)') THETA(NPAR+1),
+          SE(NPAR+1),GAMMA
END IF
IF (MN.EQ.1) THEN
  WRITE(*, '(F16.4,F18.4,F13.4)') THETA(NPAR+ISYM+1),
+          SE(NPAR+ISYM+1),MEAN
END IF
DO 34 I = 1, NREG
  WRITE(*, '(F16.4,F18.4,F13.4)') THETA(NPAR+ISYM+MN+I),
+          SE(NPAR+ISYM+MN+I),BX(I)
34 CONTINUE
NT = 4
CALL G13FBF(NUM,NT,IP,IQ,THETA,GAMMA,CVAR,HTM,ETM,IFLAG)
WRITE (*,*)
WRITE (*, '(A,F12.4)') 'Volatility forecast = ',CVAR(NT)
WRITE (*,*)
DIST = 'T'
NREG = 2
MN = 1
DF = 4.1D0
IP = 1
IQ = 2
ISYM = 1
GAMMA = -0.2D0
NPAR = IQ + IP + 1
LWK = NREG*NUM+3*NUM+NPAR+ISYM+MN+NREG+404

```

```

PARAM(1) = 0.1D0
PARAM(2) = 0.2D0
PARAM(3) = 0.3D0
PARAM(4) = 0.4D0
PARAM(5) = 0.1D0
FCALL = .TRUE.
CALL G05CBF(SEED)
CALL G05HKF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,
+          FCALL,RVEC,IFLAG)
FCALL = .FALSE.
CALL G05HKF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,
+          FCALL,RVEC,IFLAG)
CALL G05HKF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,
+          FCALL,RVEC,IFLAG)
IFLAG = -1
DO 110 I = 1,NUM
  XTERM = ZERO
  DO 115 K = 1,NREG
    XTERM = XTERM + X(I,K)*BX(K)
115  CONTINUE
    IF (MN.EQ.1) THEN
      YT(I) = MEAN + XTERM + YT(I)
    ELSE
      YT(I) = XTERM + YT(I)
    END IF
110 CONTINUE
  CALL E04UEF('Nolist')
  CALL E04UEF('Print Level = 0')
  COPTS(1) = .TRUE.
  COPTS(2) = .TRUE.
  MAXIT = 200
  TOL = 1.0D-16
  DO 112 I = 1,NPAR
    THETA(I) = PARAM(I)*0.5D0
112 CONTINUE
  THETA(NPAR+ISYM) = GAMMA*0.5D0
  THETA(NPAR+ISYM+1) = DF*0.5D0
  CALL G13FAF(DIST,YT,X,LDX,NUM,IP,IQ,NREG,MN,ISYM,
+          THETA,SE,SC,COVAR,NPARMX,HP,ETM,HTM,LGF,
+          COPTS,MAXIT,TOL,WK,LWK,IFLAG)
  WRITE(*,*)
  WRITE(*,*)'Student t-distribution'
  WRITE(*,*)
  WRITE(*,*)
+ 'Parameter estimates Standard errors Correct values'
  DO 133 I = 1, NPAR
    WRITE(*, '(F16.4,F18.4,F13.4)') THETA(I),
+   SE(I),PARAM(I)
133 CONTINUE
  IF (ISYM.EQ.1) THEN
    WRITE(*, '(F16.4,F18.4,F13.4)') THETA(NPAR+ISYM),
+   SE(NPAR+ISYM),GAMMA
  END IF
  WRITE(*, '(F16.4,F18.4,F13.4)') THETA(NPAR+ISYM+1),
+   SE(NPAR+ISYM+1),DF
  IF (MN.EQ.1) THEN
    WRITE(*, '(F16.4,F18.4,F13.4)') THETA(NPAR+ISYM+1+MN),
+   SE(NPAR+ISYM+1+MN),MEAN
  END IF
  DO 134 I = 1, NREG
    WRITE(*, '(F16.4,F18.4,F13.4)') THETA(NPAR+ISYM+1+MN+I),
+   SE(NPAR+ISYM+1+MN+I),BX(I)
134 CONTINUE
199 CONTINUE
  NT = 4
  CALL G13FBF(NUM,NT,IP,IQ,THETA,GAMMA,CVAR,HTM,ETM,IFLAG)
  WRITE (*,*)
  WRITE (*, '(A,F12.4)') 'Volatility forecast = ',CVAR(NT)
  END

```

## Output results

G13FAF Example Program Results

Gaussian distribution

Parameter estimates	Standard errors	Correct values
0.8031	0.0788	0.8000
0.6249	0.0570	0.6000
0.1803	0.0327	0.2000
0.0921	0.0237	0.1000
-0.5119	0.0682	-0.4000
2.9860	0.0324	3.0000

Volatility forecast = 2.8040

Student t-distribution

Parameter estimates	Standard errors	Correct values
0.0871	0.0230	0.1000
0.2174	0.0488	0.2000
0.2736	0.0820	0.3000
0.3588	0.0788	0.4000
-0.3240	0.0598	-0.2000
4.5173	0.5128	4.1000
3.0182	0.0431	3.0000
1.4727	0.0265	1.5000
2.4640	0.0302	2.5000

Volatility forecast = 0.4133

## 4.2 AGARCH-type2

In this example the following two models are considered:

- An AGARCH(1,1)-type2 model with shocks from a Gaussian distribution and observations,  
$$y_t = b_0 + x_t^1 b_1 + x_t^2 b_2 + e_t$$
- An AGARCH(1,1)-type2 model with shocks from a Student's  $t$ -distribution and observations,  $y_t = b_0 + x_t^1 b_1 + x_t^2 b_2 + e_t$

Sequences of 1500 observations are generated for both of these processes, and then modelled using initial parameter estimates of half the true values. The final model parameter estimates are then output and a four step ahead volatility forecast is calculated.

## Fortran source code

```
INTEGER NPARMX,NUM
DOUBLE PRECISION ZERO
PARAMETER (NPARMX=10,NUM=1500,ZERO=0.0D0)
INTEGER NUM1,MXNT,NREGMX,NT
PARAMETER (NUM1=3000,MXNT=400,NREGMX=10)
DOUBLE PRECISION FAC1,GAMMA,HP,LGF,MEAN,TOL,XTERM
INTEGER I,IFLAG,IP,IQ,K,LDX,LWK,MAXIT,MN,NPAR,NREG,SEED
LOGICAL FCALL
DOUBLE PRECISION BX(10),COVAR(NPARMX,NPARMX),ETM(NUM1),
+ HT(NUM1+10),HTM(NUM1),PARAM(NPARMX),
+ RVEC(40),SC(NPARMX),SE(NPARMX),THETA(NPARMX),
+ WK(NUM1*3+NPARMX+NREGMX*NUM1+20*20+1),X(NUM1,10),
+ YT(NUM1+10),CVAR(100)
LOGICAL COPTS(2)
CHARACTER*1 DIST
DOUBLE PRECISION DF
EXTERNAL E04UEF,G05HLF,G13FCF,G13FDF,G05CBF
INTRINSIC ABS,DBLE,SIN
```

```

WRITE(*,*)'G13FCF Example Program Results'
SEED = 111
LDX = NUM1
BX(1) = 1.5D0
BX(2) = 2.5D0
BX(3) = 3.0D0
MEAN = 3.0D0
DO 5 I = 1,NUM
    FAC1 = DBLE(I)*0.01D0
    X(I,1) = 0.01D0 + 0.7D0*SIN(FAC1)
    X(I,2) = 0.5D0 + FAC1*0.1D0
    X(I,3) = 1.0D0
5 CONTINUE
MN = 1
NREG = 2
GAMMA = -0.4D0
IP = 1
IQ = 1
NPAR = IQ + IP + 1
LWK = NREG*NUM+3*NUM+NPAR+NREG+MN+404
PARAM(1) = 0.08D0
PARAM(2) = 0.2D0
PARAM(3) = 0.7D0
FCALL = .TRUE.
CALL G05CBF(SEED)
DIST = 'N'
DF = 4.1D0
CALL G05HLF(DIST,300,IP,IQ,PARAM,GAMMA,DF,HT,YT,
+          FCALL,RVEC,IFLAG)
FCALL = .FALSE.
CALL G05HLF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,
+          FCALL,RVEC,IFLAG)
DO 110 I = 1,NUM
    XTERM = ZERO
    DO 120 K = 1,NREG
        XTERM = XTERM + X(I,K)*BX(K)
120 CONTINUE
    IF (MN.EQ.1) THEN
        YT(I) = MEAN + XTERM + YT(I)
    ELSE
        YT(I) = XTERM + YT(I)
    END IF
110 CONTINUE
IFLAG = -1
DO 130 I = 1,NPAR
    THETA(I) = PARAM(I)*0.5D0
130 CONTINUE
THETA(NPAR+1) = GAMMA*0.5D0
IF (MN.EQ.1) THEN
    THETA(NPAR+1+MN) = MEAN*0.5D0
END IF
DO 135 I = 1,NREG
    THETA(NPAR+1+MN+I) = BX(I)*0.5D0
135 CONTINUE
CALL E04UEF('Nolist')
CALL E04UEF('Print Level = 0')
MAXIT = 50
TOL = 1.0D-12
COPTS(1) = .TRUE.
COPTS(2) = .TRUE.
CALL G13FCF(DIST,YT,X,LDX,NUM,IP,IQ,NREG,MN,THETA,
+          SE,SC,COVAR,NPARMX,
+          HP,ETM,HTM,LGF,COPTS,MAXIT,TOL,WK,LWK,IFLAG)
WRITE(*,*)
WRITE(*,*)'Gaussian distribution'
WRITE(*,*)
WRITE(*,*)'Parameter estimates Standard errors Correct values'
DO 33 I = 1, NPAR
    WRITE(*, '(F16.4,F18.4,F16.4)') THETA(I),SE(I),PARAM(I)
33 CONTINUE
WRITE(*, '(F16.4,F18.4,F16.4)') THETA(NPAR+1),SE(NPAR+1),GAMMA
IF (MN.EQ.1) THEN
    WRITE(*, '(F16.4,F18.4,F16.4)') THETA(NPAR+2),
+ SE(NPAR+2),MEAN
END IF
DO 34 I = 1, NREG
    WRITE(*, '(F16.4,F18.4,F16.4)') THETA(NPAR+MN+1+I),
+ SE(NPAR+MN+1+I),BX(I)
34 CONTINUE

```

```

NT = 4
CALL G13FDF(NUM,NT,IP,IQ,THETA,GAMMA,CVAR,HTM,ETM,IFLAG)
WRITE (*,*)
WRITE (*,'(A,F12.4)') 'Volatility forecast = ',CVAR(NT)
WRITE (*,*)
LWK = NUM1*3 + NPARMX + NREGMX*NUM1 + 1
LDX = NUM1

BX(1) = 1.5D0
BX(2) = 2.5D0
BX(3) = 3.0D0
MEAN = 3.0D0
DO 25 I = 1,NUM
    FAC1 = DBLE(I)*0.01D0
    X(I,1) = 0.01D0 + 0.7D0*SIN(FAC1)
    X(I,2) = 0.5D0 + FAC1*0.1D0
    X(I,3) = 1.0D0
25 CONTINUE
MN = 1
NREG = 2
GAMMA = -0.4D0
IP = 1
IQ = 1
NPAR = IQ + IP + 1
LWK = NREG*NUM+3*NUM+NPAR+NREG+MN+405
PARAM(1) = 0.1D0
PARAM(2) = 0.1D0
PARAM(3) = 0.8D0
FCALL = .TRUE.
CALL G05CBF(SEED)
DIST = 'T'
CALL G05HLF(DIST,300,IP,IQ,PARAM,GAMMA,DF,HT,YT,
+          FCALL,RVEC,IFLAG)
FCALL = .FALSE.
CALL G05HLF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,
+          FCALL,RVEC,IFLAG)
FCALL = .FALSE.
CALL G05HLF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,
+          FCALL,RVEC,IFLAG)
DO 111 I = 1,NUM
    XTERM = ZERO
    DO 121 K = 1,NREG
        XTERM = XTERM + X(I,K)*BX(K)
121 CONTINUE
    IF (MN.EQ.1) THEN
        YT(I) = MEAN + XTERM + YT(I)
    ELSE
        YT(I) = XTERM + YT(I)
    END IF
111 CONTINUE
IFLAG = -1
DO 131 I = 1,NPAR
    THETA(I) = PARAM(I)*0.5D0
131 CONTINUE
THETA(NPAR+1) = GAMMA*0.5D0
THETA(NPAR+2) = DF*0.5D0
IF (MN.EQ.1) THEN
    THETA(NPAR+2+MN) = MEAN*0.5D0
END IF
DO 235 I = 1,NREG
    THETA(NPAR+MN+2+I) = BX(I)*0.5D0
235 CONTINUE
CALL E04UEF('Nolist')
CALL E04UEF('Print Level = 0')
MAXIT = 100
TOL = 1.0D-12
COPTS(1) = .TRUE.
COPTS(2) = .TRUE.
CALL G13FCF(DIST,YT,X,LDX,NUM,IP,IQ,NREG,MN,THETA,SE,SC,COVAR,
+          NPARMX,HP,ETM,HTM,LGF,COPTS,MAXIT,TOL,WK,LWK,IFLAG)
WRITE(*,*)
WRITE(*,*)'Student t-distribution'
WRITE(*,*)
WRITE(*,*)'Parameter estimates Standard errors Correct values'
DO 133 I = 1, NPAR
    WRITE(*,'(F16.4,F18.4,F16.4)') THETA(I),SE(I),PARAM(I)
133 CONTINUE
WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+1),SE(NPAR+1),GAMMA
WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+2),SE(NPAR+2),DF

```

```

      IF (MN.EQ.1) THEN
        WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+2+MN),
+       SE(NPAR+2+MN),MEAN
      END IF

      DO 134 I = 1, NREG
        WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+2+MN+I),
+       SE(NPAR+2+MN+I),BX(I)
134  CONTINUE
      NT = 4
      CALL G13FDF(NUM,NT,IP,IQ,THETA,GAMMA,CVAR,HTM,ETM,IFLAG)
      WRITE (*,*)
      WRITE (*,'(A,F12.4)') 'Volatility forecast = ',CVAR(NT)
      END

```

## Output results

G13FCF Example Program Results

Gaussian distribution

Parameter estimates	Standard errors	Correct values
0.0835	0.0154	0.0800
0.2150	0.0312	0.2000
0.6896	0.0324	0.7000
-0.3757	0.0655	-0.4000
3.0453	0.0591	3.0000
1.4567	0.0389	1.5000
2.4572	0.0445	2.5000

Volatility forecast = 3.0383

Student t-distribution

Parameter estimates	Standard errors	Correct values
0.0945	0.0364	0.1000
0.0800	0.0264	0.1000
0.8197	0.0523	0.8000
-0.5142	0.1418	-0.4000
3.7504	0.3687	4.1000
3.0045	0.0631	3.0000
1.5321	0.0378	1.5000
2.4799	0.0471	2.5000

Volatility forecast = 2.3701

## 4.3 GJR-GARCH

In this example the following two models are considered:

- A GJR-GARCH(1,1) model with shocks from a Gaussian distribution and observations,
$$y_t = b_0 + x_t^1 b_1 + x_t^2 b_2 + e_t$$
- A GJR-GARCH(1,1) model with shocks from a Student's  $t$ -distribution and observations,
$$y_t = b_0 + x_t^1 b_1 + x_t^2 b_2 + e_t$$

Sequences of 2000 observations are generated for both of these processes, and then modelled using initial parameter estimates of half the true values. The final model parameter estimates are then output and a four step ahead volatility forecast is calculated.

## Fortran source code

```
INTEGER NPARMX,NUM
DOUBLE PRECISION ZERO
PARAMETER (NPARMX=10,NUM=2000,ZERO=0.0D0)
INTEGER NUM1,MXNT,NREGMX,NT
PARAMETER (NUM1=3000,MXNT=400,NREGMX=10)
DOUBLE PRECISION FAC1,GAMMA,HP,LGF,MEAN,TOL,XTERM
INTEGER I,IFLAG,IP,IQ,K,LDX,LWK,MAXIT,MN,NPAR,NREG,SEED
LOGICAL FCALL
DOUBLE PRECISION BX(10),COVAR(NPARMX,NPARMX),ETM(NUM1),
+           HT(NUM1+10),HTM(NUM1),PARAM(NPARMX),
+           RVEC(40),SC(NPARMX),SE(NPARMX),THETA(NPARMX),
+           WK(NUM1*3+NPARMX+NREGMX*NUM1+20*20+1),X(NUM1,10),
+           YT(NUM1+10),CVAR(100)
LOGICAL COPTS(2)
CHARACTER*1 DIST
DOUBLE PRECISION DF
EXTERNAL E04UEF,G05HMF,G13FEF,G13FFF,G05CBF
INTRINSIC ABS,DBLE,SIN

WRITE(*,*)'G13FEF Example Program Results'
SEED = 111
LWK = NUM1*3 + NPARMX + NREGMX*NUM1 + 1
NREG = 0
LDX = NUM1
DF = 5.1D0
GAMMA = 0.1D0
BX(1) = 1.5D0
BX(2) = 2.5D0
BX(3) = 3.0D0
MEAN = 4.0D0
DO 5 I = 1,NUM
    FAC1 = DBLE(I)*0.01D0
    X(I,2) = 0.01D0 + 0.7D0*SIN(FAC1)
    X(I,1) = 0.5D0 + FAC1*0.1D0
    X(I,3) = 1.0D0
5 CONTINUE
MN = 1
NREG = 2
GAMMA = 0.1D0
IP = 1
IQ = 1
NPAR = IQ + IP + 1
PARAM(1) = 0.4D0
PARAM(2) = 0.1D0
PARAM(3) = 0.7D0
FCALL = .TRUE.
DIST = 'N'
CALL G05CBF(SEED)
CALL G05HMF(DIST,200,IP,IQ,PARAM,GAMMA,DF,
+           HT,YT,FCALL,RVEC,IFLAG)
FCALL = .FALSE.
CALL G05HMF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,
+           HT,YT,FCALL,RVEC,IFLAG)
DO 76 I = 1,NUM
    XTERM = ZERO
    DO 77 K = 1,NREG
        XTERM = XTERM + X(I,K)*BX(K)
77 CONTINUE
    IF (MN.EQ.1) THEN
        YT(I) = MEAN + XTERM + YT(I)
    ELSE
        YT(I) = XTERM + YT(I)
    END IF
76 CONTINUE
IFLAG = -1
CALL E04UEF('Nolist')
CALL E04UEF('Print Level = 0')
COPTS(1) = .TRUE.
COPTS(2) = .TRUE.
MAXIT = 100
TOL = 1.0D-12
DO 81 I = 1,NPAR
    THETA(I) = PARAM(I)*0.5D0
81 CONTINUE
```



```

      THETA(NPAR+1) = GAMMA*0.5D0
      IF (MN.EQ.1) THEN
        THETA(NPAR+MN+1) = MEAN*0.5D0
      END IF
      DO 82 I = 1,NREG
        THETA(NPAR+MN+1+I) = BX(I)*0.5D0
82 CONTINUE
      CALL G13FEF(DIST,YT,X,LDX,NUM,IP,IQ,NREG,MN,THETA,
+              SE,SC,COVAR,NPARMX,
+              HP,ETM,HTM,LGF,COPTS,MAXIT,TOL,WK,LWK,IFLAG)
      WRITE(*,*)
      WRITE(*,*)'Gaussian distribution'
      WRITE(*,*)
      WRITE(*,*)'Parameter estimates Standard errors Correct values'
      DO 33 I = 1, NPAR
        WRITE(*,'(F16.4,F18.4,F16.4)') THETA(I),SE(I),PARAM(I)
33 CONTINUE
      WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+1),SE(NPAR+1),GAMMA
      IF (MN.EQ.1) THEN
        WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+2),
+      SE(NPAR+2),MEAN
      END IF
      DO 34 I = 1, NREG
        WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+MN+I+1),
+      SE(NPAR+MN+I+1),BX(I)
34 CONTINUE
      DIST = 'T'
      MEAN = 3.0D0
      DO 15 I = 1,NUM
        FAC1 = DBLE(I)*0.01D0
        X(I,2) = 0.01D0 + 0.7D0*SIN(FAC1)
        X(I,1) = 0.5D0 + FAC1*0.1D0
        X(I,3) = 1.0D0
15 CONTINUE
      NT = 4
      CALL G13FFF(NUM,NT,IP,IQ,THETA,GAMMA,CVAR,HTM,ETM,IFLAG)
      WRITE (*,*)
      WRITE (*,'(A,F12.4)') 'Volatility forecast = ',CVAR(NT)
      WRITE (*,*)
      MN = 1
      NREG = 2
      GAMMA = 0.09D0
      IP = 1
      IQ = 1
      NPAR = IQ + IP + 1
      PARAM(1) = 0.05D0
      PARAM(2) = 0.1D0
      PARAM(3) = 0.8D0
      FCALL = .TRUE.
      CALL G05CBF(SEED)
      CALL G05HMF(DIST,200,IP,IQ,PARAM,GAMMA,DF,
+              HT,YT,FCALL,RVEC,IFLAG)
      FCALL = .FALSE.
      CALL G05HMF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,
+              HT,YT,FCALL,RVEC,IFLAG)
      CALL G05HMF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,
+              HT,YT,FCALL,RVEC,IFLAG)
      DO 176 I = 1,NUM
        XTERM = ZERO
        DO 177 K = 1,NREG
          XTERM = XTERM + X(I,K)*BX(K)
177 CONTINUE
        IF (MN.EQ.1) THEN
          YT(I) = MEAN + XTERM + YT(I)
        ELSE
          YT(I) = XTERM + YT(I)
        END IF
176 CONTINUE
      IFLAG = -1
      CALL E04UEF('Nolist')
      CALL E04UEF('Print Level = 0')
      MAXIT = 100
      TOL = 1.0D-14
      DO 181 I = 1,NPAR
        THETA(I) = PARAM(I)*0.5D0
181 CONTINUE
      THETA(NPAR+1) = GAMMA*0.5D0
      THETA(NPAR+2) = DF*0.5D0
      IF (MN.EQ.1) THEN

```

```

      THETA(NPAR+2+MN) = MEAN*0.5D0
    END IF
    DO 182 I = 1,NREG
      THETA(NPAR+2+MN+I) = BX(I)*0.5D0
182 CONTINUE
    COPTS(1) = .TRUE.
    COPTS(2) = .TRUE.
    CALL G13FEF(DIST,YT,X,LDX,NUM,IP,IQ,NREG,MN,THETA,SE,SC,
+    COVAR,NPARAMX, HP,ETM,HTM,LGF,COPTS,MAXIT,TOL,WK,LWK,IFLAG)
    WRITE(*,*)
    WRITE(*,*)'Student t-distribution'
    WRITE(*,*)
    WRITE(*,*)'Parameter estimates Standard errors Correct values'
    DO 133 I = 1, NPAR
      WRITE(*, '(F16.4,F18.4,F16.4)') THETA(I),SE(I),PARAM(I)
133 CONTINUE
    WRITE(*, '(F16.4,F18.4,F16.4)') THETA(NPAR+1),SE(NPAR+1),GAMMA
    WRITE(*, '(F16.4,F18.4,F16.4)') THETA(NPAR+2),SE(NPAR+2),DF
    IF (MN.EQ.1) THEN
      WRITE(*, '(F16.4,F18.4,F16.4)') THETA(NPAR+2+MN),
+ SE(NPAR+2+MN),MEAN
    END IF
    DO 134 I = 1, NREG
      WRITE(*, '(F16.4,F18.4,F16.4)') THETA(NPAR+2+MN+I),
+ SE(NPAR+2+MN+I),BX(I)
134 CONTINUE
    NT = 4
    CALL G13FFF(NUM,NT,IP,IQ,THETA,GAMMA,CVAR,HTM,ETM,IFLAG)
    WRITE (*,*)
    WRITE (*, '(A,F12.4)') 'Volatility forecast = ',CVAR(NT)
    END

```

## Output results

G13FEF Example Program Results

Gaussian distribution

Parameter estimates	Standard errors	Correct values
0.3706	0.0780	0.4000
0.1034	0.0256	0.1000
0.7080	0.0413	0.7000
0.1191	0.0370	0.1000
4.0989	0.0950	4.0000
1.4255	0.0592	1.5000
2.2613	0.0683	2.5000

Volatility forecast = 1.7056

Student t-distribution

Parameter estimates	Standard errors	Correct values
0.0377	0.0084	0.0500
0.0831	0.0229	0.1000
0.8112	0.0260	0.8000
0.1161	0.0361	0.0900
5.7626	0.6988	5.1000
2.9674	0.0363	3.0000
1.4891	0.0231	1.5000
2.5161	0.0277	2.5000

Volatility forecast = 0.5971

## 4.4 EGARCH

In this example the following two models are considered:

- An EGARCH(1,1) model with shocks from Gaussian distribution and observations,  
$$y_t = b_0 + x_t^1 b_1 + x_t^2 b_2 + e_t$$
- An EGARCH(1,2) model with shocks from a Student's  $t$ -distribution and observations,  
$$y_t = b_0 + x_t^1 b_1 + x_t^2 b_2 + e_t$$

Sequences of 2000 observations are generated for both of these processes, and then modelled using initial parameter estimates of half the true values. The final model parameter estimates are then output and a four step ahead volatility forecast is calculated.

### Fortran source code

```
INTEGER NPARMX, NUM
DOUBLE PRECISION ZERO
PARAMETER (NPARMX=10, NUM=1500, ZERO=0.0D0)
INTEGER NUM1, NREGMX, MXNT, NT
PARAMETER (NUM1=3000, NREGMX=10, MXNT=400)
DOUBLE PRECISION FAC1, HP, LGF, MEAN, TOL, XTERM
DOUBLE PRECISION DF
INTEGER I, IFLAG, IP, IQ, K, LDX, LWK, MAXIT, MN, NPAR, NREG, SEED
LOGICAL FCALL
CHARACTER*1 DIST
DOUBLE PRECISION BX(10), COVAR(NPARMX, NPARMX), ETM(NUM1),
+ HT(NUM1+10), HTM(NUM1), PARAM(NPARMX),
+ RVEC(40), SC(NPARMX), SE(NPARMX),
+ THETA(NPARMX),
+ WK(NUM1*3+NPARMX+NREGMX*NUM1+20*20+1), X(NUM1,10),
+ YT(NUM1+10), CVAR(100)
LOGICAL COPT
EXTERNAL E04UEF, G05HNF, G13FGF, G13FHF, G05CBF
INTRINSIC ABS, DBLE, SIN

WRITE(*,*) 'G13FGF Example Program Results'
SEED = 111
LDX = NUM1
BX(1) = 1.5D0
BX(2) = 2.5D0
BX(3) = 3.0D0
MEAN = 3.0D0
DO 5 I = 1, NUM
    FAC1 = DBLE(I)*0.01D0
    X(I,1) = 0.01D0 + 0.7D0*SIN(FAC1)
    X(I,2) = 0.5D0 + FAC1*0.1D0
    X(I,3) = 1.0D0
5 CONTINUE
NREG = 2
MN = 1
IP = 1
IQ = 1
NPAR = IP + 2*IQ + 1
PARAM(1) = 0.1D0
PARAM(2) = -0.3D0
PARAM(3) = 0.1D0
PARAM(4) = 0.9D0
DF = 5.0D0
DIST = 'N'
FCALL = .TRUE.
CALL G05CBF(SEED)
CALL G05HNF(DIST, 800, IP, IQ, PARAM, DF, HT, YT, FCALL,
+ RVEC, IFLAG)
FCALL = .FALSE.
CALL G05HNF(DIST, NUM, IP, IQ, PARAM, DF, HT, YT, FCALL,
+ RVEC, IFLAG)
```

```

IFLAG = -1
DO 110 I = 1,NUM
  XTERM = ZERO
  DO 115 K = 1,NREG
    XTERM = XTERM + X(I,K)*BX(K)
115  CONTINUE
    IF (MN.EQ.1) THEN
      YT(I) = MEAN + XTERM + YT(I)
    ELSE
      YT(I) = XTERM + YT(I)
    END IF
110 CONTINUE
CALL E04UEF('Nolist')
CALL E04UEF('Print Level = 0')
COPT = .TRUE.
MAXIT = 50
TOL = 1.0D-12
DO 120 I = 1,NPAR
  THETA(I) = PARAM(I)*0.5D0
120 CONTINUE
IF (MN.EQ.1) THEN
  THETA(NPAR+MN) = MEAN*0.5D0
END IF
DO 130 I = 1,NREG
  THETA(NPAR+MN+I) = BX(I)*0.5D0
130 CONTINUE
LWK = NREG*NUM + 3*NUM + 3
CALL G13FGF(DIST,YT,X,LDX,NUM,IP,IQ,NREG,MN,THETA,
+          SE,SC,COVAR,NPARMX,
+          HP,ETM,HTM,LGF,COPT,MAXIT,TOL,WK,LWK,IFLAG)
WRITE(*,*)
WRITE(*,*)'Gaussian distribution'
WRITE(*,*)
WRITE(*,*)'Parameter estimates Standard errors Correct values'
WRITE(*,*)
DO 133 I = 1, NPAR
  WRITE(*, '(F16.4,F18.4,F16.4)') THETA(I),SE(I),PARAM(I)
133 CONTINUE
IF (MN.EQ.1) THEN
  WRITE(*, '(F16.4,F18.4,F16.4)') THETA(NPAR+1),SE(NPAR+1),MEAN
END IF
DO 134 I = 1, NREG
  WRITE(*, '(F16.4,F18.4,F16.4)') THETA(NPAR+MN+I),SE(NPAR+MN+I),BX(I)
134 CONTINUE
NT = 4
CALL G13FHF(NUM,NT,IP,IQ,THETA,CVAR,HTM,ETM,IFLAG)
WRITE (*,*)
WRITE (*, '(A,F12.4)') 'Volatility forecast = ',CVAR(NT)
WRITE (*,*)
NREG = 2
MN = 1
IP = 1
IQ = 2
NPAR = IP + 2*IQ + 1
PARAM(1) = 0.1D0
PARAM(2) = -0.3D0
PARAM(3) = -0.1D0
PARAM(4) = 0.1D0
PARAM(5) = 0.3D0
PARAM(6) = 0.7D0
DF = 10.0D0
DIST = 'T'
FCALL = .TRUE.
CALL G05CBF(SEED)
CALL G05HNF(DIST,NUM,IP,IQ,PARAM,DF,HT,YT,FCALL,RVEC,IFLAG)
FCALL = .FALSE.
CALL G05HNF(DIST,NUM,IP,IQ,PARAM,DF,HT,YT,FCALL,RVEC,IFLAG)
IFLAG = -1
DO 10 I = 1,NUM
  XTERM = ZERO
  DO 15 K = 1,NREG
    XTERM = XTERM + X(I,K)*BX(K)
15  CONTINUE
    IF (MN.EQ.1) THEN
      YT(I) = MEAN + XTERM + YT(I)
    ELSE
      YT(I) = XTERM + YT(I)
    END IF
10 CONTINUE

```

```

CALL E04UEF('Nolist')
CALL E04UEF('Print Level = 0')
COPT = .TRUE.
MAXIT = 50
TOL = 1.0D-12
DO 20 I = 1,NPAR
  THETA(I) = PARAM(I)*0.5D0
20 CONTINUE
  THETA(NPAR+1) = DF*0.5D0
  IF (MN.EQ.1) THEN
    THETA(NPAR+1+MN) = MEAN*0.5D0
  END IF
  DO 30 I = 1,NREG
    THETA(NPAR+1+MN+I) = BX(I)*0.5D0
30 CONTINUE
  LWK = NREG*NUM + 3*NUM + 3
  CALL G13FGF(DIST,YT,X,LDX,NUM,IP,IQ,NREG,MN,THETA,
+           SE,SC,COVAR,NPARMX,
+           HP,ETM,HTM,LGF,COPT,MAXIT,TOL,WK,LWK,IFLAG)
  WRITE(*,*)
  WRITE(*,*)'Student t-distribution'
  WRITE(*,*)
  WRITE(*,*)'Parameter estimates  Standard errors  Correct values'
  DO 33 I = 1, NPAR
    WRITE(*, '(F16.4,F18.4,F16.4)') THETA(I),SE(I),PARAM(I)
33 CONTINUE
  WRITE(*, '(F16.4,F18.4,F16.4)') THETA(NPAR+1),SE(NPAR+1),DF
  IF (MN.EQ.1) THEN
    WRITE(*, '(F16.4,F18.4,F16.4)') THETA(NPAR+1+MN),
+   SE(NPAR+1+MN),MEAN
  END IF
  DO 34 I = 1, NREG
    WRITE(*, '(F16.4,F18.4,F16.4)') THETA(NPAR+1+MN+I),
+   SE(NPAR+1+MN+I),BX(I)
34 CONTINUE
  NT = 4
  CALL G13FHF(NUM,NT,IP,IQ,THETA,CVAR,HTM,ETM,IFLAG)
  WRITE (*,*)
  WRITE (*, '(A,F12.4)') 'Volatility forecast = ',CVAR(NT)
  END

```

## Output Results

G13FGF Example Program Results

Gaussian distribution

Parameter estimates	Standard errors	Correct values
0.1153	0.0197	0.1000
-0.3096	0.0258	-0.3000
0.1210	0.0382	0.1000
0.8937	0.0141	0.9000
2.8624	0.0796	3.0000
1.4518	0.0445	1.5000
2.5815	0.0534	2.5000

Volatility forecast = 2.9468

Student t-distribution

Parameter estimates	Standard errors	Correct values
0.0992	0.0363	0.1000
-0.2467	0.0548	-0.3000
-0.1361	0.0529	-0.1000
0.0670	0.0846	0.1000
0.3498	0.0680	0.3000
0.7298	0.0543	0.7000
7.9630	1.4374	10.0000
2.9934	0.0734	3.0000
1.4935	0.0535	1.5000
2.4933	0.0564	2.5000

Volatility forecast = 1.4868

## 5 Testing the software

The method used here to check whether the GARCH parameter estimation software has been implemented correctly is to use it to model known GARCH sequences.

In each test a known regression-GARCH( $p,q$ ) process is defined for a fixed parameter vector  $\mathbf{q}$ . Monte Carlo simulation is then used to repeatedly generate many different GARCH sequences and compute the mean and variance of the  $\mathbf{q}$  estimates. In each of the tests:

- the initial parameter estimates are taken to be half the true values
- only GARCH(1,2) sequences are modelled

The time dependent regression vector  $x_t$  has components in all the tests has the form:

- a constant component, 1
- a linear ramp component,  $\frac{1}{2} + \frac{t}{1000}$
- a sinusoidal component,  $\frac{1}{100} + 0.7 \sin\left(\frac{t}{100}\right)$

The difficulty of modelling a GARCH( $p,q$ ) sequence depends on both  $p$  and  $q$  and also on how much *volatility memory* there is in the process. Higher values of the  $\mathbf{b}_i$  parameters give rise to more volatility memory and are therefore harder to model accurately. Increasing the number of model parameters will also make the model more difficult to model simply because there are more variables to numerically optimise. This suggests the following order of difficulty ARCH(1), ARCH(2), ARCH(3), GARCH(1,1), GARCH(1,2), GARCH(2,2),...,etc.

It can be seen that a GARCH(1,2) model therefore represents a *reasonably* difficult test of the software.

As previously mentioned, the simulations are used to calculate the mean of the parameter estimates  $\mathbf{q}$  and the mean of the estimated standard errors. These values are then compared with the true model parameter values and the actual standard errors of the parameter values.

The simulation results are presented in tables 1-20 of Section 6. The first column labelled "Estimated Value" refers to the average parameter estimate using either 300 or 200 simulations. The second column labelled "Estimated Standard Error" refers to the average of the standard errors computed by the GARCH software. The third column labelled "Standard Error of Estimates" refers to the actual standard error of the parameter estimates.

# 6 Monte Carlo results

## AGARCH-type1

The AGARCH(1,2)-type1 simulations used here had the following parameters:

$$\begin{aligned}
 k &= 3 \\
 \mathbf{a}_0 &= 0.2, \mathbf{a}_1 = 0.1, \mathbf{a}_2 = 0.15, \mathbf{b}_1 = 0.7 \\
 \mathbf{g} &= -0.2 \\
 \mathbf{b}_0 &= 0, \mathbf{b}_1 = -1.5, \mathbf{b}_2 = 2.5, \mathbf{b}_3 = -3.0 \\
 x_t^1 &= \frac{1}{100} + 0.7 \sin\left(\frac{t}{100}\right) \\
 x_t^2 &= \frac{1}{2} + \frac{t}{1000} \\
 x_t^3 &= 1
 \end{aligned}$$

The series shocks were taken from either a Gaussian distribution or a Student's  $t$ -distribution, with the number of degrees of freedom ( $df$ ) set to 4.10.

The GARCH model parameters are output in the descending order,  $\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2, \mathbf{b}_1, \mathbf{g}$ , then  $df$  (if present) followed by  $b_i, i = 1, \dots, 3$

### Gaussian distribution

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2267	0.1712	0.1147	0.20
0.0873	0.0581	0.0607	0.10
0.1483	0.0903	0.0816	0.15
0.6944	0.1046	0.0793	0.70
-0.2408	0.3024	0.2398	-0.20
-1.4982	0.2586	0.2472	-1.50
2.5562	0.8595	0.7728	2.50
-3.0455	0.6588	0.5912	-3.00

Table 1: 400 observations and 300 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2022	0.0669	0.0639	0.20
0.0924	0.0411	0.0396	0.10
0.1468	0.0572	0.0526	0.15
0.7050	0.0517	0.0473	0.70
-0.2109	0.1598	0.1380	-0.20
-1.5072	0.0962	0.0971	-1.50
2.5011	0.1619	0.1571	2.50
-3.0001	0.1695	0.1651	-3.00

Table 2: 1000 observations and 200 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2038	0.0467	0.0442	0.20
0.0959	0.0285	0.0283	0.10
0.1526	0.0395	0.0378	0.15
0.6986	0.0357	0.0332	0.70
-0.2076	0.0991	0.0944	-0.20
-1.5004	0.0708	0.0661	-1.50
2.4993	0.0576	0.0559	2.50
-2.9998	0.0952	0.0902	-3.00

Table 3: 2000 observations and 200 simulations



## Student's *t*-distribution

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1983	0.0706	0.0676	0.20
0.0937	0.0496	0.0592	0.10
0.1528	0.0684	0.0765	0.15
0.6970	0.0586	0.0582	0.70
-0.2321	0.1954	0.1850	-0.20
4.3246	0.6645	0.6189	4.10
-1.5000	0.0765	0.0812	-1.50
2.4907	0.1270	0.1274	2.00
-2.9950	0.1331	0.1354	-3.00

Table 4: 1000 observations and 200 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2011	0.0463	0.0456	0.20
0.0975	0.0369	0.0337	0.10
0.1523	0.0485	0.0472	0.15
0.6954	0.0379	0.0394	0.70
-0.2203	0.1338	0.1213	-0.20
4.2278	0.4241	0.4069	4.10
-1.5064	0.0524	0.0517	-1.50
2.4974	0.0451	0.0439	2.00
-2.9987	0.0743	0.0709	-3.00

Table 5: 2000 observations and 200 simulations

## AGARCH-type2

The AGARCH(1,2)-type2 simulations used here had the following parameters:

$$\begin{aligned}
 k &= 3 \\
 \mathbf{a}_0 &= 0.1, \mathbf{a}_1 = 0.1, \mathbf{a}_2 = 0.15, \mathbf{b}_1 = 0.7 \\
 \mathbf{g} &= -0.1 \\
 b_0 &= 0, b_1 = -1.5, b_2 = 2.5, b_3 = -3.0 \\
 x_t^1 &= \frac{1}{100} + 0.7 \sin\left(\frac{t}{100}\right) \\
 x_t^2 &= \frac{1}{2} + \frac{t}{1000} \\
 x_t^3 &= 1
 \end{aligned}$$

The series shocks were taken from either a Gaussian distribution or a Student's  $t$ -distribution, with the number of degrees of freedom ( $df$ ) set to 4.10.

The GARCH model parameters are output in the descending order,  $\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2, \mathbf{b}_1, \mathbf{g}$ , then  $df$  (if present) followed by  $b_i, i = 1, \dots, 3$

### Gaussian distribution

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1238	0.0651	0.0572	0.10
0.1020	0.0636	0.0627	0.10
0.1459	0.0847	0.0856	0.15
0.6778	0.0914	0.0845	0.70
-0.1097	0.1298	0.1116	-0.10
-1.5245	0.1900	0.1763	-1.50
2.4532	0.6022	0.5550	2.50
-2.9564	0.4586	0.4268	-3.00

Table 6: 400 observations and 300 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1081	0.0324	0.0315	0.10
0.1015	0.0436	0.0402	0.10
0.1482	0.0587	0.0538	0.15
0.6918	0.0441	0.0487	0.70
-0.0993	0.0669	0.0613	-0.10
-1.5029	0.0678	0.0675	-1.50
2.4971	0.1083	0.1089	2.50
-2.9935	0.1116	0.1142	-3.00

Table 7: 1000 observations and 200 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1048	0.0219	0.0215	0.10
0.1004	0.0310	0.0285	0.10
0.1491	0.0402	0.0379	0.15
0.6960	0.0329	0.0333	0.70
-0.1017	0.0461	0.0425	-0.10
-1.5006	0.0473	0.0459	-1.50
2.4991	0.0356	0.0391	2.50
-2.9996	0.0571	0.0628	-3.00

Table 8: 2000 observations and 200 simulations

## Student's $t$ -distribution

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1103	0.0397	0.0344	0.10
0.0984	0.0548	0.0568	0.10
0.1547	0.0746	0.0752	0.15
0.6856	0.0666	0.0598	0.70
-0.1060	0.0942	0.0848	-0.10
4.2912	0.6639	0.6037	4.10
-1.4998	0.0460	0.0540	-1.50
2.5087	0.0853	0.0882	2.00
-3.0118	0.0860	0.0931	-3.00

Table 9: 1000 observations and 200 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1035	0.0236	0.0221	0.10
0.1005	0.0360	0.0347	0.10
0.1528	0.0488	0.0484	0.15
0.6924	0.0417	0.0404	0.70
-0.0998	0.0599	0.0569	-0.10
4.1854	0.4345	0.3988	4.10
-1.4971	0.0324	0.0360	-1.50
2.4986	0.0284	0.0306	2.00
-2.9977	0.0441	0.0494	-3.00

Table 10: 2000 observations and 200 simulations

# GJR-GARCH

The GJR-GARCH(1,2) simulations used here had the following parameters:

$$\begin{aligned}
 k &= 3 \\
 \mathbf{a}_0 &= 0.1, \mathbf{a}_1 = 0.15, \mathbf{a}_2 = 0.2, \mathbf{b}_1 = 0.4 \\
 \mathbf{g} &= 0.1 \\
 \mathbf{b}_0 &= 0, \mathbf{b}_1 = -1.5, \mathbf{b}_2 = 2.5, \mathbf{b}_3 = -3.0 \\
 x_t^1 &= \frac{1}{100} + 0.7 \sin\left(\frac{t}{100}\right) \\
 x_t^2 &= \frac{1}{2} + \frac{t}{1000} \\
 x_t^3 &= 1
 \end{aligned}$$

The series shocks were taken from either a Gaussian distribution or a Student's  $t$ -distribution, with the number of degrees of freedom ( $df$ ) set to 4.10.

The GARCH model parameters are output in the descending order,  $\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2, \mathbf{b}_1, \mathbf{g}$ , then  $df$  (if present) followed by  $b_i, i = 1, \dots, 3$

## Gaussian Distribution

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1103	0.0410	0.0364	0.10
0.1367	0.0766	0.0774	0.15
0.2003	0.1185	0.1045	0.20
0.3805	0.1435	0.1270	0.40
0.1006	0.0766	0.0759	0.10
-1.4983	0.1010	0.0992	-1.50
2.4982	0.3288	0.3119	2.50
-3.0025	0.2560	0.2397	-3.00

Table 11: 400 observations and 300 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1048	0.0269	0.0221	0.10
0.1451	0.0537	0.0502	0.15
0.1997	0.0752	0.0669	0.20
0.3936	0.0927	0.0784	0.40
0.0958	0.0511	0.0478	0.10
-1.5038	0.0379	0.0387	-1.50
2.4975	0.0645	0.0627	2.50
-2.9970	0.0677	0.0659	-3.00

Table 12: 1000 observations and 200 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1018	0.0172	0.0152	0.10
0.1470	0.0387	0.0358	0.15
0.2001	0.0499	0.0476	0.20
0.3972	0.0602	0.0548	0.40
0.1007	0.0385	0.0340	0.10
-1.5016	0.0261	0.0264	-1.50
2.4998	0.0229	0.0224	2.50
-3.0000	0.0363	0.0361	-3.00

Table 13: 2000 observations and 200 simulations

## Student's $t$ -distribution

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1027	0.0266	0.0285	0.10
0.1392	0.0691	0.0813	0.15
0.2047	0.0932	0.1028	0.20
0.3904	0.0976	0.1131	0.40
0.1158	0.0655	0.1067	0.10
4.2716	0.6603	0.6395	4.10
-1.5013	0.0328	0.0336	-1.50
2.4977	0.0526	0.0564	2.00
-2.9989	0.0549	0.0599	-3.00

Table 14: 1000 observations and 200 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1008	0.0178	0.0170	0.10
0.1469	0.0501	0.0531	0.15
0.2016	0.0716	0.0685	0.20
0.3940	0.0683	0.0663	0.60
0.1067	0.0489	0.0606	0.10
4.2076	0.4360	0.4040	4.10
-1.4982	0.0221	0.0231	-1.50
2.4994	0.0187	0.0188	2.00
-3.0003	0.0299	0.0314	-3.00

Table 15: 2000 observation and 200 simulations

# EGARCH

The EGARCH(1,2) simulations used here had the following parameters:

$$k = 3, \mathbf{a}_0 = 0.3, \mathbf{a}_1 = -0.2, \mathbf{a}_2 = -0.25,$$

$$\mathbf{f}_1 = 0.1, \mathbf{f}_2 = 0.15, \mathbf{b}_1 = 0.3,$$

$$b_0 = 0, b_1 = 1.5, b_2 = 2.5, b_3 = 3.0$$

$$x_t^2 = \frac{1}{100} + 0.7 \sin\left(\frac{t}{100}\right)$$

$$x_t^1 = \frac{1}{2} + \frac{t}{1000}$$

$$x_t^3 = 1$$

The series shocks were taken from either a Gaussian distribution or a Student's  $t$ -distribution, with the number of degrees of freedom ( $df$ ) set to 4.10.

The GARCH model parameters are output in the descending order,  $\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2, \mathbf{f}_1, \mathbf{f}_2, \mathbf{b}_1$ , then  $df$  (if present) followed by  $b_i, i = 1, \dots, 3$

## Gaussian Distribution

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2786	0.1016	0.0974	0.30
-0.1965	0.0758	0.0738	-0.20
-0.2481	0.0958	0.0909	-0.25
0.0699	0.1186	0.1228	0.10
0.1346	0.1445	0.1285	0.15
0.3045	0.2213	0.1841	0.30
1.5181	0.1855	0.1928	1.50
2.5622	0.6413	0.6017	2.50
2.9430	0.4929	0.4626	3.00

Table 16: 400 observations and 300 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2958	0.0709	0.0665	0.30
-0.1954	0.0471	0.0462	-0.20
-0.2533	0.0595	0.0571	-0.25
0.0848	0.0768	0.0789	0.15
0.1432	0.0850	0.0812	0.10
0.2914	0.1437	0.1247	0.30
1.4970	0.0747	0.0756	1.50
2.4969	0.1245	0.1231	2.50
3.0004	0.1301	0.1309	3.00

Table 17: 1000 observations and 200 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2982	0.0452	0.0471	0.30
-0.1980	0.0317	0.0326	-0.20
-0.2540	0.0381	0.0401	-0.25
0.0932	0.0508	0.0533	0.15
0.1491	0.0591	0.0545	0.10
0.2967	0.0895	0.0870	0.30
1.5017	0.0528	0.0515	1.50
2.4999	0.0442	0.0431	2.50
2.9984	0.0748	0.0702	3.00

Table 18: 2000 observations and 200 simulations

## Student's *t*-distribution

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2811	0.1022	0.0924	0.30
-0.2071	0.0628	0.0639	-0.20
-0.2452	0.0786	0.0794	-0.25
0.0865	0.0929	0.0903	0.10
0.1464	0.0932	0.0924	0.15
0.3223	0.1738	0.1526	0.30
4.3119	0.6819	0.4159	4.10
1.4983	0.0668	0.0738	1.50
2.4871	0.1142	0.1210	2.50
3.0094	0.1195	0.1267	3.00

Table 19: 1000 observations and 200 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2859	0.0702	0.0811	0.30
-0.2016	0.0426	0.0493	-0.20
-0.2483	0.0594	0.0706	-0.25
0.0926	0.0652	0.0657	0.10
0.1512	0.0633	0.0801	0.15
0.3154	0.1174	0.1283	0.30
4.2281	0.4348	0.3854	4.10
1.4942	0.0457	0.0539	1.50
2.4976	0.0413	0.0451	2.50
3.0001	0.0681	0.0734	3.00

Table 20: 2000 observations and 200 simulations

# Conclusions

The simulation results presented in tables 1-20 indicate that the GARCH generation and estimation Fortran 77 software performs as expected (The forecasting software has been tested elsewhere [9]). It was found that reliable parameter estimates and associated standard errors could only be achieved when at least 300 observations were included in the GARCH model. More *difficult* GARCH models (those with more parameters to estimate or higher volatility memory) were found to require even more observations to obtain consistent parameter estimates.

This report has discussed the current state of GARCH software that has been developed for Mark 20 of the NAG Fortran 77 Library. Since future modifications/improvements may take place it can only be taken as a guide to the GARCH software that will be eventually included in the NAG numerical libraries. Possible future improvements to the software could include the following:

- Changing the GARCH estimation routines so that some of the model parameters can be fixed (i.e., not estimated via maximum likelihood)
- Other non-Gaussian shocks, such as the generalised error distribution, etc.
- Other univariate GARCH models such GARCH-M etc.
- Multivariate GARCH models
- Generalising the GARCH model from regression-GARCH( $p,q$ ) to ARMA( $p1,q1$ )-GARCH( $p2,q2$ )

Other possible modifications, concerned with language/user-interface, are as follows:

- The inclusion of some modern Fortran features, such as memory allocation, to improve the user-interface. Since Fortran 77 does not support memory allocation the current GARCH software requires the user to calculate how much workspace is needed. The routines also only allow up to 20 parameters to be estimated; this should be sufficient for most requirements. The NAG C GARCH software, which uses internal memory allocation, does not have these restrictions and is therefore easier to use.
- The creation of Visual Basic wrappers which perform memory allocation and have optional parameters etc. These could, for example, take the form of a GARCH Microsoft Excel add-in which would have the advantage of interactive use and also shield users from the raw Fortran 77 code.
- The creation of a C++ class that contains all the GARCH functions, has optional/default function parameters and also hides workspace, etc.

To conclude, it should be mentioned that the PC and workstation implementations of the NAG numerical libraries permit software developers to call GARCH functions more easily than the corresponding functions from time series packages. Although time series packages may offer the advantage of interactive interfaces it is not easy (or computationally efficient) to call them programmatically as a component of a larger system. The current NAG delivery mechanism is thus well suited to software developers who want to incorporate individual GARCH routines into a new application.



## 7 Acknowledgements

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All trademarks are acknowledged.

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