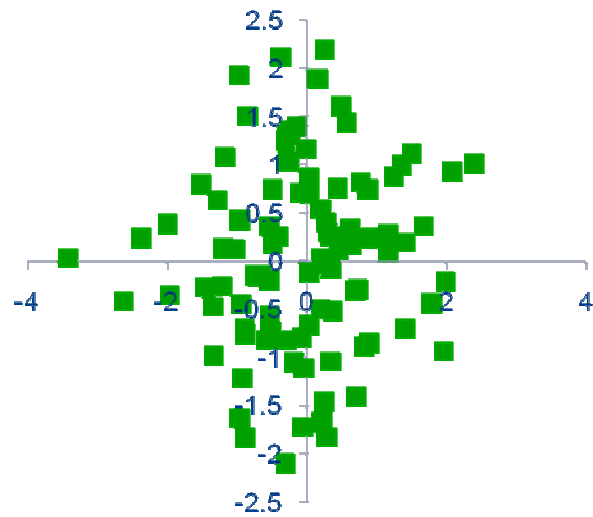


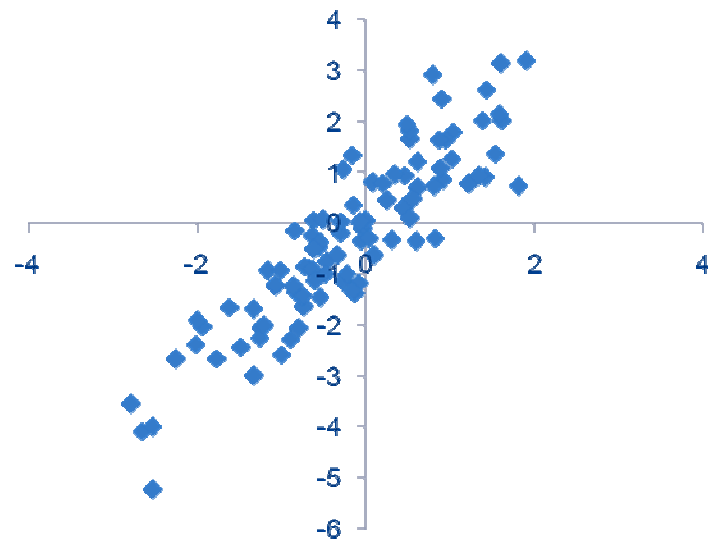
Why People Who Price Derivatives Are Interested In Correlation

Correlation Risk

What Is Correlation



No linear relationship between points



Co-movement between the points
Positive correlation

Correlation Risk Stochastics

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Standard pricing theory is based on some general stochastic description on the dynamics of the underlying asset.

With two, or more assets

$$\frac{dS_t^1}{S_t^1} = \mu^1 dt + \sigma^1 dZ^1$$

$$\frac{dS_t^2}{S_t^2} = \mu^2 dt + \sigma^2 dZ^2$$

M

$$\frac{dS_t^n}{S_t^n} = \mu^n dt + \sigma^n dZ^n$$

$$\langle dZ^i, dZ^j \rangle = \rho_{ij} dt$$

Need a correlation matrix to specify the dynamics.

Correlation Risk

Products With Correlation Exposure

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1. Quanto Option

$$N_{USD} \left(\frac{S_{JPY}^T}{S_{JPY}^0} - 1 \right)^+$$

Imagine that the share price is 100JPY and the delta is 1. In this case fund of the hedge is done in JPY. If FX rates move then the JPY of the contract moves, but the value of the hedge does not. Weak exposure to the correlation between FX and Equity

2. Compositive (Cross) Option

$$N_{USD} \left(\frac{X_{USD}^{JPY} S_{JPY}^T}{X_{USD}^{JPY} S_{JPY}^0} - 1 \right)^+$$

Like an option on a ADR. Every time we trade delta we fund in USD. Volatility is the volatility of S measured in USD. Strong exposure to FX and Equity correlation.

Correlation Risk

Products With Correlation Exposure

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1. Spread Option

$$\left(\frac{S_T^1}{S_0^1} - \frac{S_T^2}{S_0^2} - K \right)^+$$

2. Best-of, Worst-Of options

$$\left(\text{Max}_i \left(\frac{S_T^i}{S_0^i} \right) - K \right)^+ \quad \left(\text{Min}_i \left(\frac{S_T^i}{S_0^i} \right) - K \right)^+$$

3. Basket Options

$$\left(\sum_i w_i \left(\frac{S_T^i}{S_0^i} \right) - K \right)^+$$

4. Himalayan Options

$$\left(\sum_i w_i \text{Max}_{\text{remaining } i} \left(\frac{S_{t_i}^i}{S_0^i} \right) - K \right)^+$$

5. Rainbow Options

$$\left(w_1 \text{Max}_i \left(\frac{S_T^i}{S_0^i} \right) + w_2 \text{Min}_i \left(\frac{S_T^i}{S_0^i} \right) - K \right)^+$$

Correlation Risk Basket Options

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- Very commonly traded
- Can be traded with the OTC market together with short positions in options on each of the underlyings – a correlation product
- Approximations for basket variance

$$\sigma_{basket}^2 \approx \sum_{i,j} w_i w_j \rho_{ij} \sigma_i \sigma_j$$

(Note this is only an approximation as assets are **log**normally distributed.

- Increasing the correlation increases the basket volatility and makes the options more expensive
- Often sold on diverse set of underlyings to make the option cheaper
- More underlyings in the mix makes the option cheaper.

Correlation Risk

Best-Of / Worst-Of

- General idea is that when the correlation is low (negative) there is a more diverse range of outcomes, hence there will always be one asset which has outperformed and one that has under performed. (Consider the case of perfect negative correlation.)
- Following exposure of product to a rise in correlation

	Call	Put
Best-Of	-	+
Worst-Of	+	-

- Frequently components of structured products
 - Worst of Call (embedded in a guaranteed product) correlation used to make the product cheaper
 - Short Worst of Put (embedded in reverse convertible) correlation used to increase the coupon

Correlation Risk

Himalayan / Rainbow

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- Himalayan is like a combination of a basket options and an asian option - averaging over both time and asset. “Best” assets are fixed early and lose their time value. Becomes like a call on the worst – positive exposure to correlation
- Rainbow is an interesting product.
Like a basket increasing correlation, increases the value of the product
Typically the product is set up with the highest weight applied to the best performing asset, and so has some features of a call on the best – increasing correlation, decreases the value of the product
Overall product has small correlation exposure, but it can be either positive, or negative.

Correlation Risk

Typically Investment Bank Exposure

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Typical products that investment banks sell.

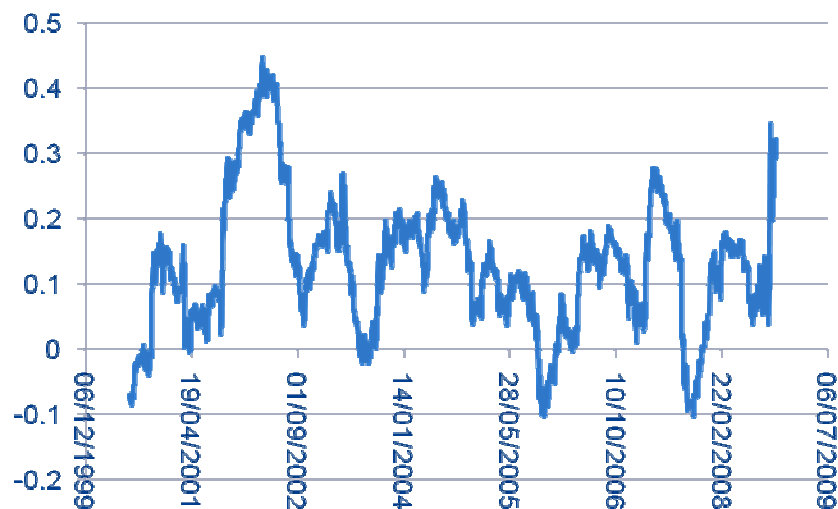
- Reverse Convertible on the worst
- Basket options
- Himalayan / Rainbow
- Calls on worst

All leave the seller short correlation. Difficult to manufacture a product which has correlation exposure in the other direction.

Correlation Risk

Measuring Correlation

First attempt would be to use the time series of two underlyings and measure the correlation of the time series. Rolling window of 6 months daily data Nikkei 225 and S&P 500



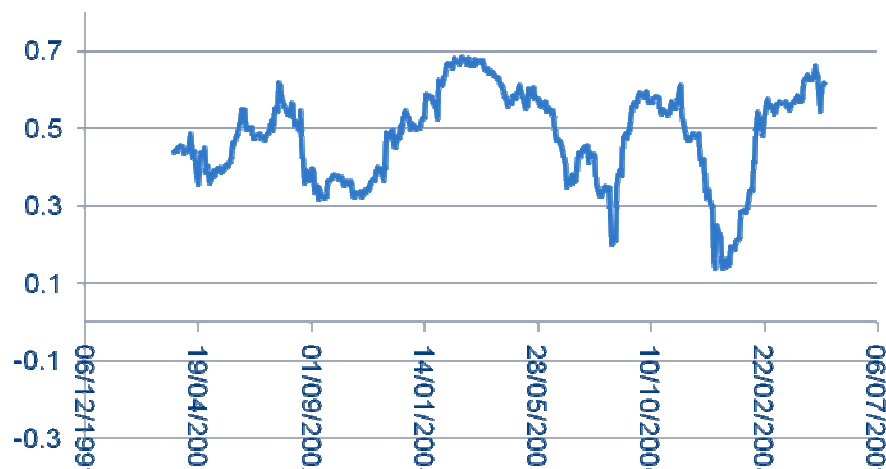
Note enormous range. For $\rho=0.2$ and 6 months daily data – statistic confidence interval is $[0.02, 0.36]$. Are we picking up sampling error, or uncertainty in correlation

Correlation is low due to asynchronous effect.

Correlation Risk

Measuring Correlation (Weekly Data)

Using a 1 year rolling window



Implied correlation taken from options markets close to 0.70.
Just as volatility trades at a premium to historic, so does correlation.

Need a mechanism that assess this premium.

Correlation Risk

Why Do We Need a Positive Semi Definite Matrices

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Negative Definite matrix implies that there are portfolios with negative variance.
(In portfolio theory can we get away with this if we insist that all assets have positive weight ?)

Monte Carlo Path Generation

Given uncorrelated random numbers W_j

How do we construct random numbers Z_j which have the correlation matrix P

Find pseudo-square root Q so that $R = QQ^T$ and $Z_j = \sum_k q_{jk} W_k$

Algorithms for Q require R to be positive semi-definite.

Correlation Risk

Why Do We Get Negative Definite Matrices

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One would imagine that if we use time series over the same period and estimated from this the correlation matrix would be positive definite by definition.

- Data is not synchronous. Different regions have different holidays.
- What do you do with the stock that has only just been issued.

In reality correlation matrices suffer from this problem.

For pricing purposes would prefer to use correlations implied from option contracts that I can see. For example baskets and dispersions.

Most likely going to be mixing historic and implied estimates.

Many people want to know what happens when correlation goes up by 10%.

Correlation Risk

Correlation Premium

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Useful transform

$$\rho \rightarrow \rho + \alpha(1 - \rho)$$

If covariance matrix is positive semi-definite then this transform will maintain this property

Gives calibration method for correlation.

1. Estimate correlations by the best historical method you have available
2. From the small number of observations that you can observe estimate the correlation premium
3. Apply this correlation premium to all historic correlations that you use.

Correlation Risk

Correlation Smile

Consider an Index such as the EuroStoxx made up of 50 underlying assets $I = \sum_i w_i S_i$

Then to reasonable approximation

$$\sigma_I^2 = \sum_{i,j} w_i w_j \rho_{ij} \sigma_i \sigma_j$$

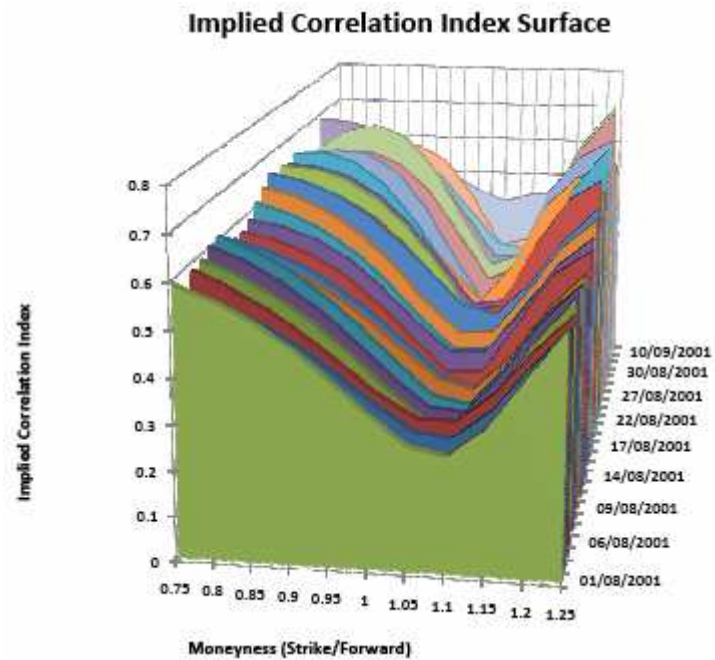
If volatility is a function of (percentage) strike then ρ must also be a function of this strike.

$$\sigma_I^2(K) = \sum_{i,j} w_i w_j \rho_{ij}(K) \sigma_i(K) \sigma_j(K)$$

We know that index volatility smiles are steeper than single stock volatility smiles so not surprisingly we find that correlation is higher for low strikes than for higher strikes

Correlation Risk

Implied Correlation



Correlation Risk

Explanation of Correlation Smile

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1. We know that distributions of assets are not lognormal. To make more sense of the information we should be using the implied distributions of each stock.
2. Given the distributions of each asset does not determine the joint distribution of two assets. There is a extensive theory of this called copula.
3. If we used local volatility as a process we would get very different results.

Correlation smile means many different things to different people.

If trading correlation the most objective way of doing so will be with variance swaps.

Correlation Risk

Products Exposed to Correlation Smile

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- Dispersion of deep out of the money options
- Far out of the money
 - Worst of options
 - Worst of digitals
- Way out of the money altiplano products
- Worst of equity default products

Correlation Risk

Measuring Explicit Correlation Risk

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- Common to measure correlation risk as

$$\rho \rightarrow \rho + 10\%$$

- What about correlations at 0.95
- What happens if the correlation matrix is no longer positive definite

- Use the α risk methodology

$$\rho \rightarrow \rho + \alpha(1 - \rho)$$

- $\alpha = 10\%$

$$\rho = 90\% \rightarrow 91\%$$

$$\rho = 20\% \rightarrow 28\%$$

- Smaller correlations move the most (in line of empirical observations)
- Maintains positive definiteness.
- Make sure that correlation scenario is sufficiently large to capture any non-linearity in correlation.

Correlation Risk

Hidden Correlation Risk

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- Correlation exposure within derivatives exposure is by no-means unique.
- Portfolio managers have been dealing with correlation exposure for a long time
- Common to measure market exposure by moving all assets simultaneously.
 - All the same amount
 - Scaled by a beta to the market
- If portfolio only had two assets which were negatively correlated then this risk measure would be in-appropriate
- Portfolio risk systems such as Barra break down risk exposure into other factors.
 - Named factors related to real variables
 - Principal components
- If you have the break down of delta by asset, worthwhile thinking about whether portfolio manager techniques can help better understand the correlation of a portfolio whether it has derivatives in it, or not.

Correlation Risk

Hedging Correlation Exposure

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Most retail structured products leave the seller short correlation.

Reverse Cliquet on a Basket

$$\text{Max} \left[X - \sum_t \left(1 - \frac{B_t}{B_{t-1}} \right)^+, 0 \right]$$

Reverse Convertible on a Basket

$$\text{Min} \left[\frac{B_T}{B_0}, 1 \right]$$

Small number of products which leave the seller long correlation.

Correlation Risk

Hedging Correlation Risk - Dispersion

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Broker market traders dispersions in vanilla options

$$\text{Max} \left[w_1 \frac{S_T^1}{S_0^1} + w_2 \frac{S_T^2}{S_0^2} - K, 0 \right] - w_1 \text{Max} \left[\frac{S_T^1}{S_0^1} - K, 0 \right] - w_2 \text{Max} \left[\frac{S_T^2}{S_0^2} - K, 0 \right]$$

Can go long or short and hence long, or short correlation.

On initiation product has little individual vega, but has exposure to correlation.

- When share prices move this will no longer be the case.
- Just as the vega of a vanilla option dissipates when the option move away from ATM, so the correlation exposure of a basket option dissipates as you move away from ATM.

These types of trades are popular with indices.

Correlation Risk

Hedging Correlation Exposure – Variance Swaps

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Variance Swaps provide a simple way of trading variance without an explicit strike dependency.

Can trade a dispersion of variance swaps

Long variance swap on the EuroStoxx 50

Short variance swaps on each of the individual constituents.

Can go long or short.

- Possible on the indices with a small number of assets, but can be approximated on indices such as the S&P with tracking portfolios
- As these are strike-less the correlation exposure does not dissipate as the underlying assets move.

Correlation Risk

Correlation Swaps

A correlation swap is an OTC product which has a payoff given by

$$P = \frac{2}{N(N-1)} \sum_{i < j} \rho_{ij}$$

Gives a direct way of trading correlation.

- Difficult to hedge product
- Does it really give the exposure you require.

If you mis-estimate correlation between two Geometric Brownian motions

P&L given by terms including

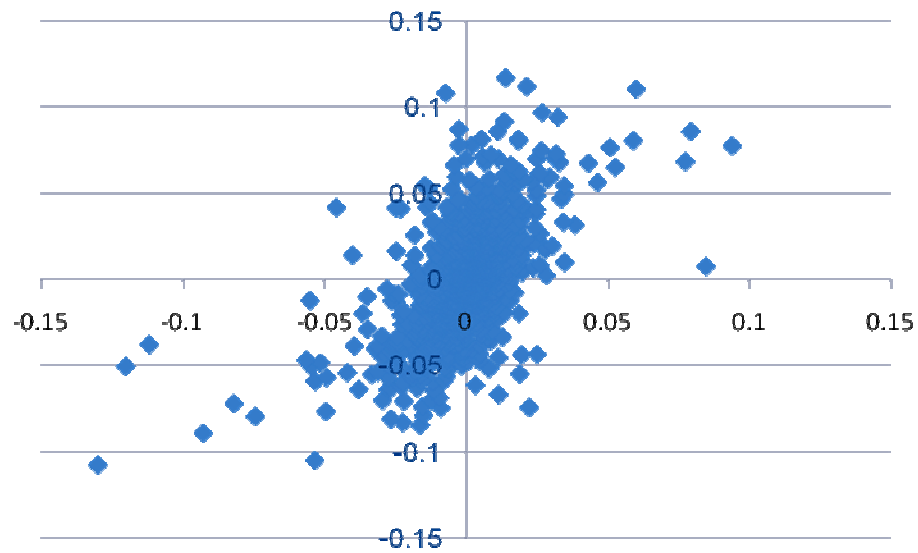
$$E \left[\int_0^T \frac{\partial^2 f}{\partial S_1 \partial S_2} S_1 S_2 \sigma_1 \sigma_2 (\hat{\rho}_{ij} - \rho_{ij}) dt \right]$$

Covariance swaps will be easier to price.

Correlation Risk

Return to Q-Q Maps

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Is assumption correlation constant correct.
Should we be using a copula based theory.