

NAG Library Function Document

nag_opt_one_var_no_deriv (e04abc)

1 Purpose

nag_opt_one_var_no_deriv (e04abc) searches for a minimum, in a given finite interval, of a continuous function of a single variable, using function values only. The method (based on quadratic interpolation) is intended for functions which have a continuous first derivative (although it will usually work if the derivative has occasional discontinuities).

2 Specification

```
#include <nag.h>
#include <nage04.h>

void nag_opt_one_var_no_deriv (void (*funct)(double xc, double *fc,
    Nag_Comm *comm),
    double e1, double e2, double *a, double *b, Integer max_fun, double *x,
    double *f, Nag_Comm *comm, NagError *fail)
```

3 Description

nag_opt_one_var_no_deriv (e04abc) is applicable to problems of the form:

$$\text{Minimize } F(x) \quad \text{subject to } a \leq x \leq b.$$

It normally computes a sequence of x values which tend in the limit to a minimum of $F(x)$ subject to the given bounds. It also progressively reduces the interval $[a, b]$ in which the minimum is known to lie. It uses the safeguarded quadratic-interpolation method described in Gill and Murray (1973).

You must supply a function **funct** to evaluate $F(x)$. The arguments **e1** and **e2** together specify the accuracy

$$Tol(x) = \mathbf{e1} \times |x| + \mathbf{e2}$$

to which the position of the minimum is required. Note that **funct** is never called at any point which is closer than $Tol(x)$ to a previous point.

If the original interval $[a, b]$ contains more than one minimum, nag_opt_one_var_no_deriv (e04abc) will normally find one of the minima.

4 References

Gill P E and Murray W (1973) Safeguarded steplength algorithms for optimization using descent methods *NPL Report NAC 37* National Physical Laboratory

5 Arguments

- 1: **funct** – function, supplied by the user *External Function*
funct must calculate the value of $F(x)$ at any point x in $[a, b]$.

The specification of **funct** is:

```
void funct (double xc, double *fc, Nag_Comm *comm)
```

1: **xc** – double

Input

On entry: x , the point at which the value of $F(x)$ is required.

2:	fc – double *	<i>Output</i>
	<i>On exit:</i> the value of the function F at the current point x .	
3:	comm – Nag_Comm *	
	Pointer to structure of type Nag_Comm; the following members are relevant to funct .	
	first – Nag_Boolean	<i>Input</i>
	<i>On entry:</i> will be set to Nag_TRUE on the first call to funct and Nag_FALSE for all subsequent calls.	
	nf – Integer	<i>Input</i>
	<i>On entry:</i> the number of calls made to funct so far.	
	user – double *	
	iuser – Integer *	
	p – Pointer	
	The type Pointer will be void * with a C compiler that defines void * and char * otherwise. Before calling nag_opt_one_var_no_deriv (e04abc) these pointers may be allocated memory and initialized with various quantities for use by funct when called from nag_opt_one_var_no_deriv (e04abc).	

Note: **funct** should be tested separately before being used in conjunction with nag_opt_one_var_no_deriv (e04abc).

- 2: **e1** – double *Input*
On entry: the relative accuracy to which the position of a minimum is required. (Note that since **e1** is a relative tolerance, the scaling of x is automatically taken into account.)
It is recommended that **e1** should be no smaller than 2ϵ , and preferably not much less than $\sqrt{\epsilon}$, where ϵ is the *machine precision*.
If **e1** is set to a value less than ϵ , its value is ignored and the default value of $\sqrt{\epsilon}$ is used instead. In particular, you may set **e1** = 0.0 to ensure that the default value is used.
- 3: **e2** – double *Input*
On entry: the absolute accuracy to which the position of a minimum is required. It is recommended that **e2** should be no smaller than 2ϵ .
If **e2** is set to a value less than ϵ , its value is ignored and the default value of $\sqrt{\epsilon}$ is used instead. In particular, you may set **e2** = 0.0 to ensure that the default value is used.
- 4: **a** – double * *Input/Output*
On entry: the lower bound a of the interval containing a minimum.
On exit: an improved lower bound on the position of the minimum.
- 5: **b** – double * *Input/Output*
On entry: the upper bound b of the interval containing a minimum.
On exit: an improved upper bound on the position of the minimum.
Constraint: **b** > **a** + **e2**.
Note that the value **e2** = $\sqrt{\epsilon}$ applies here if **e2** < ϵ on entry to nag_opt_one_var_no_deriv (e04abc).

- 6: **max_fun** – Integer *Input*
On entry: the maximum number of function evaluations (calls to **funct**) which you are prepared to allow.
 The number of evaluations actually performed by `nag_opt_one_var_no_deriv` (e04abc) may be determined by supplying a non-NULL argument **comm** (see below) and examining the structure member **comm** → **nf** on exit.
Constraint: **max_fun** ≥ 3.
 (Few problems will require more than 30 function evaluations.)
- 7: **x** – double * *Output*
On exit: the estimated position of the minimum.
- 8: **f** – double * *Output*
On exit: the value of F at the final point **x**.
- 9: **comm** – Nag_Comm * *Input/Output*
Note: **comm** is a NAG defined type (see Section 3.2.1.1 in the Essential Introduction).
On entry/exit: structure containing pointers for communication to user-supplied functions; see the above description of **funct** for details. The number of times the function **funct** was called is returned in the member **comm** → **nf**.
 If you do not need to make use of this communication feature, the null pointer `NAGCOMM_NULL` may be used in the call to `nag_opt_one_var_no_deriv` (e04abc); **comm** will then be declared internally for use in calls to user-supplied functions.
- 10: **fail** – NagError * *Input/Output*
 The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_2_REAL_ARG_GE

On entry, $\mathbf{a} + \mathbf{e2} = \langle \text{value} \rangle$ while $\mathbf{b} = \langle \text{value} \rangle$. These arguments must satisfy $\mathbf{a} + \mathbf{e2} < \mathbf{b}$.

NE_INT_ARG_LT

On entry, **max_fun** must not be less than 3: **max_fun** = $\langle \text{value} \rangle$.

NW_MAX_FUN

The maximum number of function calls, $\langle \text{value} \rangle$, have been performed.

This may have happened simply because **max_fun** was set too small for a particular problem, or may be due to a mistake in the user-supplied function, **funct**. If no mistake can be found in **funct**, restart `nag_opt_one_var_no_deriv` (e04abc) (preferably with the values of **a** and **b** given on exit from the previous call to `nag_opt_one_var_no_deriv` (e04abc)).

7 Accuracy

If $F(x)$ is δ -unimodal for some $\delta < Tol(x)$, where $Tol(x) = \mathbf{e1} \times |x| + \mathbf{e2}$, then, on exit, x approximates the minimum of $F(x)$ in the original interval $[a, b]$ with an error less than $3 \times Tol(x)$.

8 Further Comments

Timing depends on the behaviour of $F(x)$, the accuracy demanded, and the length of the interval $[a, b]$. Unless $F(x)$ can be evaluated very quickly, the run time will usually be dominated by the time spent in **funct**.

If $F(x)$ has more than one minimum in the original interval $[a, b]$, `nag_opt_one_var_no_deriv` (e04abc) will determine an approximation x (and improved bounds a and b) for one of the minima.

If `nag_opt_one_var_no_deriv` (e04abc) finds an x such that $F(x - \delta_1) > F(x) < F(x + \delta_2)$ for some $\delta_1, \delta_2 \geq Tol(x)$, the interval $[x - \delta_1, x + \delta_2]$ will be regarded as containing a minimum, even if $F(x)$ is less than $F(x - \delta_1)$ and $F(x + \delta_2)$ only due to rounding errors in the user-supplied function. Therefore **funct** should be programmed to calculate $F(x)$ as accurately as possible, so that `nag_opt_one_var_no_deriv` (e04abc) will not be liable to find a spurious minimum.

9 Example

A sketch of the function

$$F(x) = \frac{\sin x}{x}$$

shows that it has a minimum somewhere in the range $[3.5, 5.0]$. The example program below shows how `nag_opt_one_var_no_deriv` (e04abc) can be used to obtain a good approximation to the position of a minimum.

9.1 Program Text

```

/* nag_opt_one_var_no_deriv (e04abc) Example Program.
 *
 * Copyright 1998 Numerical Algorithms Group.
 *
 * Mark 5, 1998.
 * Mark 7 revised, 2001.
 * Mark 8 revised, 2004.
 *
 */

#include <nag.h>
#include <nagx04.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <math.h>
#include <nage04.h>

#ifdef __cplusplus
extern "C" {
#endif
static void NAG_CALL funct(double xc, double *fc, Nag_Comm *comm);
#ifdef __cplusplus
}
#endif

static void NAG_CALL funct(double xc, double *fc, Nag_Comm *comm)
{
    *fc = sin(xc) / xc;
}
/* funct */

int main(int argc, char *argv[])
{
    FILE      *fpout;
    Integer    exit_status = 0, max_fun;
    NagError  fail;
    Nag_Comm  comm;
    double     a, b, e1, e2, f, x;

```

```

INIT_FAIL(fail);

/* Check for command-line IO options */
fpout = nag_example_file_io(argc, argv, "-results", NULL);

fprintf(fpout,
        "nag_opt_one_var_no_deriv (e04abc) Example Program Results\n\n");

/* e1 and e2 are set to zero so that nag_opt_one_var_no_deriv (e04abc) will
 * reset them to their default values.
 */
e1 = 0.0;
e2 = 0.0;
/* The minimum is known to lie in the range (3.5, 5.0) */
a = 3.5;
b = 5.0;
/* Allow 30 calls of funct */
max_fun = 30;
/* nag_opt_one_var_no_deriv (e04abc).
 * Minimizes a function of one variable, using function values only.
 */
nag_opt_one_var_no_deriv(funcnt, e1, e2, &a, &b, max_fun, &x, &f, &comm,
                        &fail);
if (fail.code != NE_NOERROR)
{
    fprintf(fpout, "Error from nag_opt_one_var_no_deriv (e04abc).\n%s\n",
            fail.message);
    exit_status = 1;
    goto END;
}

fprintf(fpout, "The minimum lies in the interval %7.5f to %7.5f.\n", a, b);
fprintf(fpout, "Its estimated position is %7.5f,\n", x);
fprintf(fpout, "where the function value is %13.4e.\n", f);
fprintf(fpout, "%lld function evaluations were required.\n", comm.nf);

END:
if (fpout != stdout) fclose(fpout);
return exit_status;
}

```

9.2 Program Data

None.

9.3 Program Results

nag_opt_one_var_no_deriv (e04abc) Example Program Results

The minimum lies in the interval 4.49341 to 4.49341.
 Its estimated position is 4.49341,
 where the function value is -2.1723e-01.
 10 function evaluations were required.
