NAG Library Routine Document

F07CNF (ZGTSV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07CNF (ZGTSV) computes the solution to a complex system of linear equations

$$AX = B,$$

where A is an n by n tridiagonal matrix and X and B are n by r matrices.

2 Specification

```
SUBROUTINE F07CNF(N, NRHS, DL, D, DU, B, LDB, INFO)
INTEGER           N, NRHS, LDB, INFO
complex*16        DL(*), D(*), DU(*), B(LDB,*)
```

The routine may be called by its LAPACK name ***zgtsv***.

3 Description

F07CNF (ZGTSV) uses the Gaussian elimination with partial pivoting and row interchanges to solve the equations $AX = B$. The matrix A is factorized as $A = PLU$, where P is a permutation matrix, L is unit lower triangular with at most one nonzero subdiagonal element per column, and U is an upper triangular band matrix, with two superdiagonals.

Note that the equation $A^{\mathrm{T}}X = B$ may be solved by interchanging the order of the arguments DU and DL.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphiahttp://www.netlib.org/lapack/lug

5 Parameters

1: N – INTEGER *Input*

On entry: n, the number of linear equations, i.e., the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

2: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: $\mathrm{NRHS} \geq 0$.

3: DL($*$) – ***complex*16*** array *Input/Output*

Note: the dimension of the array DL must be at least $\max(1, \mathrm{N} - 1)$.

On entry: must contain the $(n - 1)$ subdiagonal elements of the matrix A.

On exit: if no constrains are violated, DL is overwritten by the $(n - 2)$ elements of the second superdiagonal of the upper triangular matrix U from the LU factorization of A, in $\mathrm{DL}(1), \mathrm{DL}(2), \ldots, \mathrm{DL}(n - 2)$.

4: D$(*)$ – ***complex*16*** array *Input/Output*

Note: the dimension of the array D must be at least $\max(1, \mathrm{N})$.

On entry: must contain the n diagonal elements of the matrix A.

On exit: if no constraints are violated, D is overwritten by the n diagonal elements of the upper triangular matrix U from the LU factorization of A.

5: DU$(*)$ – ***complex*16*** array *Input/Output*

Note: the dimension of the array DU must be at least $\max(1, \mathrm{N} - 1)$.

On entry: must contain the $(n - 1)$ superdiagonal elements of the matrix A.

On exit: if no constraints are violated, DU is overwritten by the $(n - 1)$ elements of the first superdiagonal of U.

6: B(LDB,$*$) – ***complex*16*** array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{NRHS})$.

To solve the equations $Ax = b$, where b is a single right-hand side, B may be supplied as a one-dimensional array with length LDB $= \max(1, \mathrm{N})$

On entry: the n by r right-hand side matrix B.

On exit: if INFO $= 0$, the n by r solution matrix X.

7: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07CNF (ZGTSV) is called.

Constraint: LDB $\geq \max(1, \mathrm{N})$.

8: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, u_{ii} is exactly zero, and the solution has not been computed. The factorization has not been completed unless $i =$ N.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

Alternatives to F07CNF (ZGTSV), which return condition and error estimates are F04CCF and F07CPF (ZGTSVX).

8 Further Comments

The total number of floating-point operations required to solve the equations $AX = B$ is proportional to nr.

The real analogue of this routine is F07CAF (DGTSV).

9 Example

This example solves the equations

$$Ax = b,$$

where A is the tridiagonal matrix

$$A = \begin{pmatrix} -1.3+1.3i & 2.0-1.0i & 0 & 0 & 0 \\ 1.0-2.0i & -1.3+1.3i & 2.0+1.0i & 0 & 0 \\ 0 & 1.0+1.0i & -1.3+3.3i & -1.0+1.0i & 0 \\ 0 & 0 & 2.0-3.0i & -0.3+4.3i & 1.0-1.0i \\ 0 & 0 & 0 & 1.0+1.0i & -3.3+1.3i \end{pmatrix}$$

and

$$b = \begin{pmatrix} 2.4-5.0i \\ 3.4+18.2i \\ -14.7+9.7i \\ 31.9-7.7i \\ -1.0+1.6i \end{pmatrix}.$$

9.1 Program Text

```
*     F07CNF Example Program Text
*     Mark 21 Release. NAG Copyright 2004.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX
      PARAMETER        (NMAX=8)
*     .. Local Scalars ..
      INTEGER          I, INFO, N
*     .. Local Arrays ..
      COMPLEX *16      B(NMAX), D(NMAX), DL(NMAX-1), DU(NMAX-1)
*     .. External Subroutines ..
      EXTERNAL         ZGTSV
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07CNF Example Program Results'
      WRITE (NOUT,*)
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*        Read the tridiagonal matrix A and the right hand side B from
*        data file
*
         READ (NIN,*) (DU(I),I=1,N-1)
         READ (NIN,*) (D(I),I=1,N)
         READ (NIN,*) (DL(I),I=1,N-1)
         READ (NIN,*) (B(I),I=1,N)
*
*        Solve the equations Ax = b for x
```

```
*
         CALL ZGTSV(N,1,DL,D,DU,B,N,INFO)
*
         IF (INFO.EQ.0) THEN
*
*           Print solution
*
            WRITE (NOUT,*) 'Solution'
            WRITE (NOUT,99999) (B(I),I=1,N)
*
         ELSE
            WRITE (NOUT,99998) 'The (', INFO, ',', INFO, ')',
     +        ' element of the factor U is zero'
         END IF
       ELSE
         WRITE (NOUT,*) 'NMAX too small'
       END IF
*
99999 FORMAT (4(' (',F8.4,',',F8.4,')',:))
99998 FORMAT (1X,A,I3,A,I3,A,A)
      END
```

9.2 Program Data

```
F07CNF Example Program Data
   5                                                        :Value of N
 (  2.0, -1.0) (  2.0,  1.0) ( -1.0,  1.0) (  1.0, -1.0) :End of DU
 ( -1.3,  1.3) ( -1.3,  1.3) ( -1.3,  3.3) ( -0.3,  4.3)
 ( -3.3,  1.3)                                            :End of D
 (  1.0, -2.0) (  1.0 , 1.0) (  2.0, -3.0) (  1.0,  1.0) :End of  DL
 (  2.4, -5.0) (  3.4, 18.2) (-14.7,  9.7) ( 31.9, -7.7)
 ( -1.0,  1.6)                                            :End of B
```

9.3 Program Results

```
 F07CNF Example Program Results

 Solution
 (  1.0000,  1.0000) (  3.0000, -1.0000) (  4.0000,  5.0000) ( -1.0000, -2.0000)
 (  1.0000, -1.0000)
```