1 Purpose

s01ba returns a value of the shifted logarithmic function, \( \ln(1 + x) \), via the function name.

2 Syntax

\[
\begin{align*}
\text{[result, ifail]} &= \text{s01ba}(x) \\
\end{align*}
\]

3 Description

s01ba computes values of \( \ln(1 + x) \), retaining full relative precision even when \( |x| \) is small. The function is based on the Chebyshev expansion

\[
\ln \frac{1 + p^2 + 2px}{1 + p^2 - 2px} = 4 \sum_{k=0}^{\infty} \frac{2^k+1}{2^k + 1} T_{2k+1}(\bar{x}).
\]

Setting \( \bar{x} = \frac{x(1 + p^2)}{2p(x + 2)} \), and choosing \( p = \frac{q - 1}{q + 1} \) \( q = \sqrt{2} \) the expansion is valid in the domain

\[
x \in \left[ \frac{1}{\sqrt{2}} - 1, \sqrt{2} - 1 \right].
\]

Outside this domain, \( \ln(1 + x) \) is computed by the standard logarithmic function.

4 References


5 Parameters

5.1 Compulsory Input Parameters

1: \( x \) – double scalar

The argument \( x \) of the function.

Constraint: \( x > -1.0 \).

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: \( \text{result} \) – double scalar

The result of the function.

2: \( \text{ifail} \) – int32 scalar

\( \text{ifail} = 0 \) unless the function detects an error (see Section 6).
6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1
   On entry, $x \leq -1.0$.
   The result is returned as zero.

7 Accuracy

The returned result should be accurate almost to *machine precision*, with a limit of about 20 significant figures due to the precision of internal constants. Note however that if $x$ lies very close to $-1.0$ and is not exact (for example if $x$ is the result of some previous computation and has been rounded), then precision will be lost in the computation of $1 + x$, and hence $\ln(1 + x)$, in s01ba.

8 Further Comments

Empirical tests show that the time taken for a call of s01ba usually lies between about 1.25 and 2.5 times the time for a call to the standard logarithm function.

9 Example

```matlab
x = 2.5;
[result, ifail] = s01ba(x)
result =
   1.2528
ifail =
   0
```