NAG Toolbox for Matlab

s01ea

1 Purpose
s01ea evaluates the exponential function $e^z$, for complex*16 $z$.

2 Syntax

```matlab
[result, ifail] = s01ea(z)
```

3 Description
s01ea evaluates the exponential function $e^z$, taking care to avoid machine overflow, and giving a warning if the result cannot be computed to more than half precision. The function is evaluated as $e^z = e^x (\cos y + i \sin y)$, where $x$ and $y$ are the real and imaginary parts respectively of $z$.

Since $\cos y$ and $\sin y$ are less than or equal to 1 in magnitude, it is possible that $e^x$ may overflow although $e^x \cos y$ or $e^x \sin y$ does not. In this case the alternative formula $\text{sign}(\cos y)e^{x + \ln|\cos y|}$ is used for the real part of the result, and $\text{sign}(\sin y)e^{x + \ln|\sin y|}$ for the imaginary part. If either part of the result still overflows, a warning is returned through parameter `ifail`.

If Im$(z)$ is too large, precision may be lost in the evaluation of $\sin y$ and $\cos y$. Again, a warning is returned through `ifail`.

4 References
None.

5 Parameters

5.1 Compulsory Input Parameters

1: $z$ – complex scalar

The argument $z$ of the function.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: `result` – complex scalar

The result of the function.

2: `ifail` – int32 scalar

`ifail = 0` unless the function detects an error (see Section 6).
6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

The real part of the result overflows, and is set to the largest safe number with the correct sign. The imaginary part of the result is meaningful.

ifail = 2

The imaginary part of the result overflows, and is set to the largest safe number with the correct sign. The real part of the result is meaningful.

ifail = 3

Both real and imaginary parts of the result overflow, and are set to the largest safe number with the correct signs.

ifail = 4

The computed result is accurate to less than half precision, due to the size of Im(z).

ifail = 5

The computed result has no precision, due to the size of Im(z), and is set to zero.

7 Accuracy

Accuracy is limited in general only by the accuracy of the standard functions in the computation of \( \sin y \), \( \cos y \) and \( e^x \), where \( x = \text{Re}(z) \), \( y = \text{Im}(z) \). As \( y \) gets larger, precision will probably be lost due to argument reduction in the evaluation of the sine and cosine functions, until the warning error ifail = 4 occurs when \( y \) gets larger than \( \sqrt{1/\epsilon} \), where \( \epsilon \) is the machine precision. Note that on some machines, the intrinsic functions SIN and COS will not operate on arguments larger than about \( \sqrt{1/\epsilon} \), and so ifail can never return as 4.

In the comparatively rare event that the result is computed by the formulae

\[
sign(\cos y)e^{x+\ln|\cos y|}
\]

and

\[
sign(\sin y)e^{x+\ln|\sin y|}
\]

a further small loss of accuracy may be expected due to rounding errors in the logarithmic function.

8 Further Comments

None.

9 Example

```matlab
z = complex(1, +0);
[result, ifail] = s01ea(z)
```

result =

\[ 2.7183 \]

ifail =

\[ 0 \]