NAG Toolbox for Matlab

s10ab

1 Purpose
s10ab returns the value of the hyperbolic sine, sinh \( x \), via the function name.

2 Syntax
\[
[result, ifail] = s10ab(x)
\]

3 Description
s10ab calculates an approximate value for the hyperbolic sine of its argument, sinh \( x \).
For \( |x| \leq 1 \) it uses the Chebyshev expansion
\[
sinh x = x \times y(t) = x \sum_{r=0} a_r T_r(t)
\]
where \( t = 2x^2 - 1 \).
For \( 1 < |x| \leq E_1 \), \( \sinh x = \frac{1}{2}(e^x - e^{-x}) \)
where \( E_1 \) is a machine-dependent constant.
For \( |x| > E_1 \), the function fails owing to the danger of setting overflow in calculating \( e^x \). The result returned for such calls is sinh(sign \( xE_1 \)), i.e., it returns the result for the nearest valid argument.

4 References

5 Parameters
5.1 Compulsory Input Parameters
1: \( x \) – double scalar
   The argument \( x \) of the function.

5.2 Optional Input Parameters
None.

5.3 Input Parameters Omitted from the MATLAB Interface
None.

5.4 Output Parameters
1: \( result \) – double scalar
   The result of the function.
2: \( ifail \) – int32 scalar
   ifail = 0 unless the function detects an error (see Section 6).
6 Error Indicators and Warnings

Errors or warnings detected by the function:

```matlab
ifail = 1
```

The function has been called with an argument too large in absolute magnitude. There is a danger of setting overflow. The result is the value of sinh at the closest argument for which a valid call could be made.

7 Accuracy

If \( \delta \) and \( \epsilon \) are the relative errors in the argument and result, respectively, then in principle

\[
|\epsilon| \simeq |x \coth x \times \delta|.
\]

That is the relative error in the argument, \( x \), is amplified by a factor, approximately \( x \coth x \). The equality should hold if \( \delta \) is greater than the machine precision (\( \delta \) is a result of data errors etc.) but, if \( \delta \) is simply a result of round-off in the machine representation of \( x \), then it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the error amplification factor can be seen in the following graph:

![Figure 1](image)

It should be noted that for \(|x| \geq 2\)

\[
\epsilon \sim x\delta = \Delta
\]

where \( \Delta \) is the absolute error in the argument.

8 Further Comments

None.

9 Example

```matlab
x = -10;
[result, ifail] = s10ab(x)
result =
```

s10ab.2 [NP3663/22]
ifail = 0

-1.1013e+04