1 Purpose
s11ac returns the value of the inverse hyperbolic cosine, \( \arccosh x \), via the function name. The result is in the principal positive branch.

2 Syntax

\[
\text{[result, ifail]} = \text{s11ac}(x)
\]

3 Description
s11ac calculates an approximate value for the inverse hyperbolic cosine, \( \arccosh x \). It is based on the relation

\[
\arccosh x = \ln\left(x + \sqrt{x^2 - 1}\right).
\]

This form is used directly for \( 1 < x < 10^k \), where \( k = \lfloor n/2 \rfloor + 1 \), and the machine uses approximately \( n \) decimal place arithmetic.

For \( x \geq 10^k \), \( \sqrt{x^2 - 1} \) is equal to \( \sqrt{x} \) to within the accuracy of the machine and hence we can guard against premature overflow and, without loss of accuracy, calculate

\[
\arccosh x = \ln 2 + \ln x.
\]

4 References

5 Parameters

5.1 Compulsory Input Parameters
1: \( x \) – double scalar
The argument \( x \) of the function.
Constraint: \( x \geq 1.0 \).

5.2 Optional Input Parameters
None.

5.3 Input Parameters Omitted from the MATLAB Interface
None.

5.4 Output Parameters
1: \( \text{result} \) – double scalar
The result of the function.

2: \( \text{ifail} \) – int32 scalar
\( \text{ifail} = 0 \) unless the function detects an error (see Section 6).
6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

The function has been called with an argument less than 1.0, for which \( \text{arccosh} \, x \) is not defined. The result returned is zero.

7 Accuracy

If \( \delta \) and \( \epsilon \) are the relative errors in the argument and result respectively, then in principle

\[
|\epsilon| \simeq \left| \frac{x}{\sqrt{x^2 - 1 \text{arccosh} \, x}} \times \delta \right|
\]

That is the relative error in the argument is amplified by a factor at least \( \frac{x}{\sqrt{x^2 - 1 \text{arccosh} \, x}} \) in the result.

The equality should apply if \( \delta \) is greater than the \textit{machine precision} (\( \delta \) due to data errors etc.) but if \( \delta \) is simply a result of round-off in the machine representation it is possible that an extra figure may be lost in internal calculation and round-off. The behaviour of the amplification factor is shown in the following graph:

![Figure 1](image_url)

It should be noted that for \( x > 2 \) the factor is always less than 1.0. For large \( x \) we have the absolute error \( E \) in the result, in principle, given by

\[
E \sim \delta.
\]

This means that eventually accuracy is limited by \textit{machine precision}. More significantly for \( x \) close to 1, \( x - 1 \sim \delta \), the above analysis becomes inapplicable due to the fact that both function and argument are bounded, \( x \geq 1 \), \( \text{arccosh} \, x \geq 0 \). In this region we have

\[
E \sim \sqrt{\delta}.
\]

That is, there will be approximately half as many decimal places correct in the result as there were correct figures in the argument.
8 Further Comments

None.

9 Example

```plaintext
x = 1;
[result, ifail] = s1lac(x)

result =
  0
ifail =
  0
```