1 Purpose
s14ac returns a value of the function $\psi(x) - \ln x$, where $\psi$ is the psi function $\psi(x) = \frac{d}{dx} \ln \Gamma(x) = \frac{\Gamma'(x)}{\Gamma(x)}$.

2 Syntax

```
[result, ifail] = s14ac(x)
```

3 Description
s14ac returns a value of the function $\psi(x) - \ln x$. The psi function is computed without the logarithmic term so that when $x$ is large, sums or differences of psi functions may be computed without unnecessary loss of precision, by analytically combining the logarithmic terms. For example, the difference $d = \psi(x + \frac{1}{2}) - \psi(x)$ has an asymptotic behaviour for large $x$ given by

$$d \sim \ln(x + \frac{1}{2}) - \ln x + O\left(\frac{1}{x^2}\right) \sim \ln \left(1 + \frac{1}{2x}\right) \sim \frac{1}{2x}.$$  

Computing $d$ directly would amount to subtracting two large numbers which are close to $\ln(x + \frac{1}{2})$ and $\ln x$ to produce a small number close to $\frac{1}{2x}$, resulting in a loss of significant digits. However, using this function to compute $f(x) = \psi(x) - \ln x$, we can compute $d = f(x + \frac{1}{2}) - f(x) + \ln(1 + \frac{1}{2x})$, and the dominant logarithmic term may be computed accurately from its power series when $x$ is large. Thus we avoid the unnecessary loss of precision.

The function is derived from the function PSIFN in Amos (1983).

4 References

5 Parameters
5.1 Compulsory Input Parameters
1: x – double scalar
   The argument $x$ of the function.
   Constraint: $x > 0.0$.

5.2 Optional Input Parameters
None.

5.3 Input Parameters Omitted from the MATLAB Interface
None.
5.4 Output Parameters

1: result – double scalar
   The result of the function.

2: ifail – int32 scalar
   ifail = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1
   On entry, x ≤ 0.0. s14ac returns the value zero.

ifail = 2
   No result is computed because underflow is likely. The value of x is too large. s14ac returns the value zero.

ifail = 3
   No result is computed because overflow is likely. The value of x is too small. s14ac returns the value zero.

7 Accuracy

All constants in s14ac are given to approximately 18 digits of precision. Calling the number of digits of precision in the floating-point arithmetic being used \( t \), then clearly the maximum number of correct digits in the results obtained is limited by \( p = \min(t, 18) \).

With the above proviso, results returned by this function should be accurate almost to full precision, except at points close to the zero of \( \psi(x) \), \( x \approx 1.461632 \), where only absolute rather than relative accuracy can be obtained.

8 Further Comments

None.

9 Example

```matlab
x = 0.1;
[result, ifail] = s14ac(x)
result = -8.1212
ifail = 0
```