1 Purpose

s14ba computes values for the incomplete gamma functions $P(a, x)$ and $Q(a, x)$.

2 Syntax

[p, q, ifail] = s14ba(a, x, tol)

3 Description

s14ba evaluates the incomplete gamma functions in the normalized form

\[
P(a, x) = \frac{1}{\Gamma(a)} \int_0^x t^{a-1} e^{-t} \, dt,
\]

\[
Q(a, x) = \frac{1}{\Gamma(a)} \int_x^\infty t^{a-1} e^{-t} \, dt,
\]

with $x \geq 0$ and $a > 0$, to a user-specified accuracy. With this normalization, $P(a, x) + Q(a, x) = 1$.

Several methods are used to evaluate the functions depending on the arguments $a$ and $x$, the methods including Taylor expansion for $P(a, x)$, Legendre’s continued fraction for $Q(a, x)$, and power series for $Q(a, x)$. When both $a$ and $x$ are large, and $a \approx x$, the uniform asymptotic expansion of Temme (1987) is employed for greater efficiency – specifically, this expansion is used when $a \geq 20$ and $0.7a \leq x \leq 1.4a$.

Once either $P$ or $Q$ is computed, the other is obtained by subtraction from 1. In order to avoid loss of relative precision in this subtraction, the smaller of $P$ and $Q$ is computed first.

This function is derived from the function GAM in Gautschi (1979b).

4 References

Gautschi W (1979a) A computational procedure for incomplete gamma functions ACM Trans. Math. Software 5 466–481


5 Parameters

5.1 Compulsory Input Parameters

1:   a  – double scalar
    The argument $a$ of the functions.
    Constraint: $a > 0.0$.

2:   x  – double scalar
    The argument $x$ of the functions.
    Constraint: $x \geq 0.0$. 

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3: \( \text{tol} \) – double scalar

The relative accuracy required by you in the results. If \( \text{s14ba} \) is entered with \( \text{tol} \) greater than 1.0 or less than \( \text{machine precision} \), then the value of \( \text{machine precision} \) is used instead.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: \( p \) – double scalar

The values of the functions \( P(a, x) \) and \( Q(a, x) \) respectively.

2: \( q \) – double scalar

3: \( \text{ifail} \) – int32 scalar

\( \text{ifail} = 0 \) unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

\( \text{ifail} = 1 \)

On entry, \( a \leq 0.0 \).

\( \text{ifail} = 2 \)

On entry, \( x < 0.0 \).

\( \text{ifail} = 3 \)

Convergence of the Taylor series or Legendre continued fraction fails within 600 iterations. This error is extremely unlikely to occur; if it does, contact NAG.

7 Accuracy

There are rare occasions when the relative accuracy attained is somewhat less than that specified by parameter \( \text{tol} \). However, the error should never exceed more than one or two decimal places. Note also that there is a limit of 18 decimal places on the achievable accuracy, because constants in the function are given to this precision.

8 Further Comments

The time taken for a call of \( \text{s14ba} \) depends on the precision requested through \( \text{tol} \), and also varies slightly with the input arguments \( a \) and \( x \).

9 Example

\begin{verbatim}
 a = 2;
x = 3;
tol = 1.111307226797642e-16;
[p, q, ifail] = s14ba(a, x, tol)

p =
\end{verbatim}
\begin{verbatim}
q = 0.8009
ifail = 0
\end{verbatim}