NAG Toolbox for Matlab

s15ab

1 Purpose
s15ab returns the value of the cumulative Normal distribution function, \( P(x) \), via the function name.

2 Syntax
\[
\text{[result, ifail] = s15ab(x)}
\]

3 Description
s15ab evaluates an approximate value for the cumulative Normal distribution function
\[
P(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2} du.
\]
The function is based on the fact that
\[
P(x) = \frac{1}{2} \text{erfc}\left(\frac{-x}{\sqrt{2}}\right)
\]
and it calls s15ad to obtain a value of \( \text{erfc} \) for the appropriate argument.

4 References

5 Parameters
5.1 Compulsory Input Parameters
1: \( x \) – double scalar
   The argument \( x \) of the function.

5.2 Optional Input Parameters
None.

5.3 Input Parameters Omitted from the MATLAB Interface
None.

5.4 Output Parameters
1: \( \text{result} \) – double scalar
   The result of the function.

2: \( \text{ifail} \) – int32 scalar
   \( \text{ifail} = 0 \) unless the function detects an error (see Section 6).
Error Indicators and Warnings

There are no failure exits from this function. The parameter ifail is included for consistency with other functions in this chapter.

Accuracy

Because of its close relationship with \( \text{erfc} \), the accuracy of this function is very similar to that in \( \text{s15ad} \). If \( \epsilon \) and \( \delta \) are the relative errors in result and argument, respectively, they are in principle related by

\[
|\epsilon| \approx \frac{xe^{-\frac{1}{2}x^2}}{\sqrt{2\pi P(x)}}
\]

so that the relative error in the argument, \( x \), is amplified by a factor, \( \frac{xe^{-\frac{1}{2}x^2}}{\sqrt{2\pi P(x)}} \), in the result.

For \( x \) small and for \( x \) positive this factor is always less than one and accuracy is mainly limited by machine precision.

For large negative \( x \) the factor behaves like \( \sim x^2 \) and hence to a certain extent relative accuracy is unavoidably lost.

However the absolute error in the result, \( E \), is given by

\[
|E| \approx \frac{xe^{-\frac{1}{2}x^2}}{\sqrt{2\pi}}
\]

so absolute accuracy can be guaranteed for all \( x \).

Further Comments

None.

Example

```matlab
x = -20;
[result, ifail] = s15ab(x)
```

```
result =
   2.7536e-89
ifail =
   0
```