NAG Toolbox for Matlab

s15ae

1 Purpose
s15ae returns the value of the error function \( \text{erf}(x) \), via the function name.

2 Syntax
\[
\begin{align*}
[\text{result}, \text{ifail}] = \text{s15ae}(x)
\end{align*}
\]

3 Description
s15ae calculates an approximate value for the error function
\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \, dt = 1 - \text{erfc}(x).
\]

Let \( \hat{x} \) be the root of the equation \( \text{erfc}(x) - \text{erf}(x) = 0 \) (then \( \hat{x} \approx 0.46875 \)). For \( |x| \leq \hat{x} \) the value of \( \text{erf}(x) \) is based on the following rational Chebyshev expansion for \( \text{erf}(x) \):
\[
\text{erf}(x) \approx x R_{\ell,m}(x^2),
\]

where \( R_{\ell,m} \) denotes a rational function of degree \( \ell \) in the numerator and \( m \) in the denominator.

For \( |x| > \hat{x} \) the value of \( \text{erf}(x) \) is based on a rational Chebyshev expansion for \( \text{erfc}(x) \): for \( \hat{x} < |x| \leq 4 \) the value is based on the expansion
\[
\text{erfc}(x) \approx e^{x^2} R_{\ell,m}(x);
\]
and for \( |x| > 4 \) it is based on the expansion
\[
\text{erfc}(x) \approx e^{x^2} \left( \frac{1}{\sqrt{\pi}} + \frac{1}{x^2} R_{\ell,m}(1/x^2) \right).
\]

For each expansion, the specific values of \( \ell \) and \( m \) are selected to be minimal such that the maximum relative error in the expansion is of the order \( 10^{-d} \), where \( d \) is the maximum number of decimal digits that can be accurately represented for the particular implementation (see x02be).

For \( |x| \geq x_{\text{hi}} \) there is a danger of setting underflow in \( \text{erfc}(x) \). For \( x \geq x_{\text{hi}} \), s15ae returns \( \text{erf}(x) = 1 \); for \( x \leq -x_{\text{hi}} \) it returns \( \text{erf}(x) = -1 \).

4 References

5 Parameters
5.1 Compulsory Input Parameters
1: \( x \) – double scalar
   The argument \( x \) of the function.

5.2 Optional Input Parameters
None.
5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: result – double scalar
   The result of the function.

2: ifail – int32 scalar
   ifail = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

There are no failure exits from s15ae. The parameter ifail has been included for consistency with other functions in this chapter.

7 Accuracy

See Section 7 in s15ad.

8 Further Comments

None.

9 Example

```matlab
x = -6;
[result, ifail] = s15ae(x)
```

```
result =
  -1
ifail =
   0
```