NAG Toolbox for Matlab

s18gk

1 Purpose
s18gk returns a sequence of values for the Bessel functions $J_{\alpha+n-1}(z)$ or $J_{\alpha-n+1}(z)$ for complex $z$, non-negative $\alpha < 1$ and $n = 1, 2, \ldots, |N| + 1$.

2 Syntax
[b, ifail] = s18gk(z, a, nl)

3 Description
s18gk evaluates a sequence of values for the Bessel function of the first kind $J_{\alpha}(z)$, where $z$ is complex and nonzero and $\alpha$ is the order with $0 \leq \alpha < 1$. The $(|N|+1)$-member sequence is generated for orders $\alpha, \alpha + 1, \ldots, \alpha + |N|$ when $N \geq 0$. Note that $+$ is replaced by $-$ when $N < 0$. For positive orders the function may also be called with $z = 0$, since $J_q(0) = 0$ when $q > 0$. For negative orders the formula

$$J_{-q}(z) = \cos(\pi q)J_q(z) - \sin(\pi q)Y_q(z)$$

is used to generate the required sequence. The appropriate values of $J_q(z)$ and $Y_q(z)$ are obtained by calls to s17dc and s17de.

4 References

5 Parameters
5.1 Compulsory Input Parameters
1: z – complex scalar
   The argument $z$ of the function.
   Constraint: $z \neq (0.0, 0.0)$ when nl < 0.

2: a – double scalar
   The order $\alpha$ of the first member in the required sequence of function values.
   Constraint: $0.0 \leq a < 1.0$.

3: nl – int32 scalar
   The value of $N$.
   Constraint: abs(nl) $\leq 101$.

5.2 Optional Input Parameters
None.

5.3 Input Parameters Omitted from the MATLAB Interface
None.
5.4 Output Parameters

1: \(b(\text{abs}(nl) + 1)\) – complex array

With ifail = 0 or 3, the required sequence of function values: \(b(n)\) contains \(J_{\alpha+n-1}(z)\) if \(nl \geq 0\) and \(J_{\alpha+n+1}(z)\) otherwise, for \(n = 1, 2, \ldots, \text{abs}(nl) + 1\).

2: ifail – int32 scalar

ifail = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, \(z = (0.0, 0.0)\) when \(nl < 0\),
or \(a < 0.0\),
or \(a \geq 1.0\),
or \(\text{abs}(nl) > 101\).

ifail = 2

The computation has been abandoned due to the likelihood of overflow.

ifail = 3

The computation has been completed but some precision has been lost.

ifail = 4

The computation has been abandoned due to total loss of precision.

ifail = 5

The computation has been abandoned due to failure to satisfy the termination condition.

7 Accuracy

All constants in s17dc and s17de are specified to approximately 18 digits of precision. If \(t\) denotes the number of digits of precision in the floating-point arithmetic being used, then clearly the maximum number of correct digits in the results obtained is limited by \(p = \min(t, 18)\). Because of errors in argument reduction when computing elementary functions inside s17dc and s17de, the actual number of correct digits is limited, in general, by \(p - s\), where \(s = \max(1, \log_{10}|z|, \log_{10}|\alpha|)\) represents the number of digits lost due to the argument reduction. Thus the larger the values of \(|z|\) and \(|\alpha|\), the less the precision in the result.

8 Further Comments

None.

9 Example

```matlab
z = complex(0.6, -0.8);
a = 0;
nl = int32(3);
[b, ifail] = s18gk(z, a, nl)

b =
    1.0565 + 0.2481i
```

s18gk.2 [NP3663/22]
\begin{tabular}{l}
  0.3582 - 0.3754i \\
  -0.0260 - 0.1254i \\
  -0.0194 - 0.0086i \\
  ifail = \\
  0
\end{tabular}