1 Purpose

s20ad returns a value for the Fresnel Integral $C(x)$, via the function name.

2 Syntax

```
[result, ifail] = s20ad(x)
```

3 Description

s20ad evaluates an approximation to the Fresnel Integral

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2}t^2\right) dt.$$ \hspace{1cm}

Note: $C(x) = -C(-x)$, so the approximation need only consider $x \geq 0.0$.

The function is based on three Chebyshev expansions:

For $0 < x \leq 3$,

$$C(x) = x \sum_{r=0}^a a_r T_r(t), \quad \text{with} \quad t = 2\left(\frac{x}{3}\right)^4 - 1.$$ \hspace{1cm}

For $x > 3$,

$$C(x) = \frac{1}{2} + \frac{f(x)}{x} \sin\left(\frac{\pi}{2}x^2\right) - \frac{g(x)}{x^3} \cos\left(\frac{\pi}{2}x^2\right),$$ \hspace{1cm}

where $f(x) = \sum_{r=0}^b b_r T_r(t)$,

and $g(x) = \sum_{r=0}^c c_r T_r(t)$,

with $t = 2\left(\frac{3}{x}\right)^4 - 1$.

For small $x$, $C(x) \simeq x$. This approximation is used when $x$ is sufficiently small for the result to be correct to machine precision.

For large $x$, $f(x) \simeq \frac{1}{\pi^2}$ and $g(x) \simeq \frac{1}{\pi^2}$. Therefore for moderately large $x$, when $\frac{1}{\pi^2 x^3}$ is negligible compared with $\frac{1}{2^4}$, the second term in the approximation for $x > 3$ may be dropped. For very large $x$, when $\frac{1}{\pi^2 x}$ becomes negligible, $C(x) \simeq \frac{1}{2}$. However there will be considerable difficulties in calculating $\sin\left(\frac{\pi}{2}x^2\right)$ accurately before this final limiting value can be used. Since $\sin\left(\frac{\pi}{2}x^2\right)$ is periodic, its value is essentially determined by the fractional part of $x^2$. If $x^2 = N + \theta$, where $N$ is an integer and $0 \leq \theta < 1$, then $\sin\left(\frac{\pi}{2}x^2\right)$ depends on $\theta$ and on $N$ modulo 4. By exploiting this fact, it is possible to retain some significance in the calculation of $\sin\left(\frac{\pi}{2}x^2\right)$ either all the way to the very large $x$ limit, or at least until the integer part of $\frac{x^2}{2}$ is equal to the maximum integer allowed on the machine.
4 References

5 Parameters
5.1 Compulsory Input Parameters
1: x – double scalar
The argument $x$ of the function.

5.2 Optional Input Parameters
None.

5.3 Input Parameters Omitted from the MATLAB Interface
None.

5.4 Output Parameters
1: result – double scalar
The result of the function.
2: ifail – int32 scalar
ifail = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings
There are no failure exits from s20ad. The parameter ifail has been included for consistency with other functions in this chapter.

7 Accuracy
Let $\delta$ and $\epsilon$ be the relative errors in the argument and result respectively.

If $\delta$ is somewhat larger than the machine precision (i.e. if $\delta$ is due to data errors etc.), then $\epsilon$ and $\delta$ are approximately related by:

$$
\epsilon \approx \left| \frac{x \cos\left( \frac{\pi}{2} x^2 \right)}{C(x)} \right| \delta.
$$

Figure 1 shows the behaviour of the error amplification factor $\left| \frac{x \cos\left( \frac{\pi}{2} x^2 \right)}{C(x)} \right|$.

However, if $\delta$ is of the same order as the machine precision, then rounding errors could make $\epsilon$ slightly larger than the above relation predicts.

For small $x$, $\epsilon \simeq \delta$ and there is no amplification of relative error.

For moderately large values of $x$,

$$
|\epsilon| \approx \left| 2x \cos\left( \frac{\pi}{2} x^2 \right) \right| |\delta|
$$

and the result will be subject to increasingly large amplification of errors. However the above relation
breaks down for large values of $x$ (i.e., when $\frac{1}{x^2}$ is of the order of the machine precision); in this region the relative error in the result is essentially bounded by $\frac{2}{\pi x}$.

Hence the effects of error amplification are limited and at worst the relative error loss should not exceed half the possible number of significant figures.

Figure 1

8 Further Comments
None.

9 Example

```plaintext
x = 0;
[result, ifail] = s20ad(x)

result =
0
ifail =
0
```