NAG Toolbox for Matlab

s21be

1 Purpose

s21be returns a value of the classical (Legendre) form of the incomplete elliptic integral of the first kind, via the function name.

2 Syntax

[result, ifail] = s21be(phi, dm)

3 Description

s21be calculates an approximation to the integral

\[ F(\phi \mid m) = \int_0^{\phi} (1 - m \sin^2 \theta)^{-\frac{1}{2}} d\theta, \]

where \( 0 \leq \phi \leq \frac{\pi}{2} \), \( m \sin^2 \phi \leq 1 \) and \( m \) and \( \sin \phi \) may not both equal one.

The integral is computed using the symmetrised elliptic integrals of Carlson (Carlson (1979) and Carlson (1988)). The relevant identity is

\[ F(\phi \mid m) = \sin \phi R_F(q, r, 1), \]

where \( q = \cos^2 \phi \), \( r = 1 - m \sin^2 \phi \) and \( R_F \) is the Carlson symmetrised incomplete elliptic integral of the first kind (see s21bb).

4 References


Carlson B C (1979) Computing elliptic integrals by duplication Numerische Mathematik 33 1–16


5 Parameters

5.1 Compulsory Input Parameters

1: \( \phi \) – double scalar
2: \( m \) – double scalar

The arguments \( \phi \) and \( m \) of the function.

Constraints:

\[
0.0 \leq \phi \leq \frac{\pi}{2}; \\
\text{dm} \times \sin^2(\phi) \leq 1.0; \\
\text{Only one of } \sin(\phi) \text{ and } \text{dm} \text{ may be } 1.0.
\]

Note that \( \text{dm} \times \sin^2(\phi) = 1.0 \) is allowable, as long as \( \text{dm} \neq 1.0. \)

5.2 Optional Input Parameters

None.
5.3 Input Parameters Omitted from the MATLAB Interface
None.

5.4 Output Parameters
1:  result – double scalar
    The result of the function.

2:  ifail – int32 scalar
    ifail = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings
Errors or warnings detected by the function:

ifail = 1
    phi lies outside the range $[0, \frac{\pi}{2}]$. On soft failure, the function returns zero.

ifail = 2
    On entry, $dm \times \sin^2(\phi) > 1$; the function is undefined. On soft failure, the function returns zero.

ifail = 3
    On entry, $\sin(\phi) = 1.0$ and $dm = 1.0$; the function is infinite. On soft failure, the function returns the largest machine number (see x02al).

7 Accuracy
In principle s21be is capable of producing full machine precision. However round-off errors in internal arithmetic will result in slight loss of accuracy. This loss should never be excessive as the algorithm does not involve any significant amplification of round-off error. It is reasonable to assume that the result is accurate to within a small multiple of the machine precision.

8 Further Comments
You should consult the S Chapter Introduction, which shows the relationship between this function and the Carlson definitions of the elliptic integrals. In particular, the relationship between the argument-constraints for both forms becomes clear.

For more information on the algorithm used to compute $R_F$, see the function document for s21bb.

If you wish to input a value of $\phi$ outside the range allowed by this function you should refer to Section 17.4 of Abramowitz and Stegun (1972) for useful identities.

9 Example

```matlab
result = zeros(3, 1);
ifail = zeros(3, 1, 'int32');
fprintf('
 phi dm s21be ifail
');
for ix = 1:3
    phi = ix*pi/6;
    dm = ix/4;
    [result(ix), ifail(ix)] = s21be(phi, dm);
    fprintf(' %7.2f %7.2f %12.4f %d
', phi, dm, result(ix), ifail(ix));
end
```
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