1 Purpose

s21bj returns a value of the classical (Legendre) form of the complete elliptic integral of the second kind, via the function name.

2 Syntax

[result, ifail] = s21bj(dm)

3 Description

s21bj calculates an approximation to the integral

\[ E(m) = \int_0^\frac{\pi}{2} (1 - m \sin^2 \theta)^{\frac{1}{2}} d\theta, \]

where \( m \leq 1 \).

The integral is computed using the symmetrised elliptic integrals of Carlson (Carlson (1979) and Carlson (1988)). The relevant identity is

\[ E(m) = R_F(0, 1 - m, 1) - \frac{1}{3} m R_D(0, 1 - m, 1), \]

where \( R_F \) is the Carlson symmetrised incomplete elliptic integral of the first kind (see s21bb) and \( R_D \) is the Carlson symmetrised incomplete elliptic integral of the second kind (see s21bc).

4 References

Carlson B C (1979) Computing elliptic integrals by duplication Numerische Mathematik 33 1–16

5 Parameters

5.1 Compulsory Input Parameters

1: dm – double scalar

The argument \( m \) of the function.

Constraint: \( dm \leq 1.0 \).

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.
5.4 Output Parameters
1: result – double scalar
   The result of the function.
2: ifail – int32 scalar
   ifail = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings
Errors or warnings detected by the function:

ifail = 1
   On entry, dm > 1.0; the function is undefined. On soft failure, the function returns zero.

7 Accuracy
In principle s21bj is capable of producing full machine precision. However round-off errors in internal
arithmetic will result in slight loss of accuracy. This loss should never be excessive as the algorithm does
not involve any significant amplification of round-off error. It is reasonable to assume that the result is
accurate to within a small multiple of the machine precision.

8 Further Comments
You should consult the S Chapter Introduction, which shows the relationship between this function and the
Carlson definitions of the elliptic integrals. In particular, the relationship between the argument-constraints
for both forms becomes clear.

For more information on the algorithms used to compute $R_F$ and $R_D$, see the function documents for
s21bb and s21bc, respectively.

9 Example

```matlab
result = zeros(3, 1);
ifail = zeros(3, 1, 'int32');
fprintf('
 dm  s21bj  ifail
');
for ix = 1:3
    dm = ix/4;
    [result(ix), ifail(ix)] = s21bj(dm);
    fprintf(' %7.2f %12.4f %d
', dm, result(ix), ifail(ix));
end
```

<table>
<thead>
<tr>
<th>dm</th>
<th>s21bj</th>
<th>ifail</th>
</tr>
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<td>1.4675</td>
<td>0</td>
</tr>
<tr>
<td>0.50</td>
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<td>0.75</td>
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