NAG Toolbox for Matlab

s22aa

1 Purpose
s22aa returns a sequence of values for either the unnormalized or normalized Legendre functions of the first kind $P^m_n(x)$ or $P^m_{mn}(x)$ for real $x$ of a given order $m$ and degree $n = 0, 1, \ldots, N$.

2 Syntax

$$[p, \text{ifail}] = \text{s22aa}(\text{mode}, x, m, n)$$

3 Description
s22aa evaluates a sequence of values for either the unnormalized or normalized Legendre ($m = 0$) or associated Legendre ($m \neq 0$) functions of the first kind $P^m_n(x)$ or $P^m_{mn}(x)$, where $x$ is real with $-1 \leq x \leq 1$, of order $m$ and degree $n = 0, 1, \ldots, N$ defined by

$$P^m_n(x) = (1 - x^2)^{m/2} \frac{d^m}{dx^m} P_n(x) \quad \text{if } m \geq 0,$$

$$P^m_n(x) = \frac{(n + m)!}{(n - m)!} P^{m-m}_n(x) \quad \text{if } m < 0 \quad \text{and}$$

$$P^m_{mn}(x) = \sqrt{\frac{n + 1}{2}} \frac{(n - m)!}{(n + m)!} P^m_n(x)$$

respectively; $P_n(x)$ is the (unassociated) Legendre polynomial of degree $n$ given by

$$P_n(x) \equiv P^0_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$

(the Rodrigues formula). Note that some authors (e.g., Abramowitz and Stegun (1972)) include an additional factor of $(-1)^m$ (the Condon–Shortley Phase) in the definitions of $P^m_n(x)$ and $P^m_{mn}(x)$. They use the notation $P_{mn}(x) \equiv (-1)^m P^m_n(x)$ in order to distinguish between the two cases.

s22aa is based on a standard recurrence relation described in Section 8.5.3 of Abramowitz and Stegun (1972). Constraints are placed on the values of $m$ and $n$ in order to avoid the possibility of machine overflow. It also sets the appropriate elements of the array $p$ (see Section 5) to zero whenever the required function is not defined for certain values of $m$ and $n$ (e.g., $m = -5$ and $n = 3$).

4 References

5 Parameters

5.1 Compulsory Input Parameters

1:  \text{mode} – int32 scalar

Indicates whether the sequence of function values is to be returned unnormalized or normalized.

\text{mode} = 1

The sequence of function values is returned unnormalized.
The sequence of function values is returned normalized.

*Constraint:* \( \text{mode} = 1 \) or 2.

2: \( x \) – double scalar

The argument \( x \) of the function.

*Constraint:* \( \text{abs}(x) \leq 1.0 \).

3: \( m \) – int32 scalar

The order \( m \) of the function.

*Constraint:* \( \text{abs}(m) \leq 27 \).

4: \( nl \) – int32 scalar

The degree \( N \) of the last function required in the sequence.

*Constraints:*

\[
\begin{align*}
nl & \geq 0; \\
& \quad \text{if } m = 0, \text{ nl } \leq 100; \\
& \quad \text{if } m \neq 0, \text{ nl } \leq 55 - \text{abs}(m).
\end{align*}
\]

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: \( p(0 : nl) \) – double array

The required sequence of function values as follows:

- if \( \text{mode} = 1 \), \( p(n) \) contains \( P^m_n(x) \), for \( n = 0, 1, \ldots, N \);
- if \( \text{mode} = 2 \), \( p(n) \) contains \( \overline{P^m_n}(x) \), for \( n = 0, 1, \ldots, N \).

2: \( \text{ifail} \) – int32 scalar

\( \text{ifail} = 0 \) unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

\( \text{ifail} = 1 \)

On entry, \( \text{abs}(x) > 1.0 \),
or \( \text{mode} \neq 1 \) or 2,
or \( \text{nl} < 0 \),
or \( \text{nl} > 100 \) when \( m = 0 \),
or \( \text{abs}(m) > 27 \),
or \( \text{nl} + \text{abs}(m) > 55 \) when \( m \neq 0 \).
7 Accuracy

The computed function values should be accurate to within a small multiple of the machine precision except when underflow (or overflow) occurs, in which case the true function values are within a small multiple of the underflow (or overflow) threshold of the machine.

8 Further Comments

None.

9 Example

```matlab
mode = int32(1);
x = 0.5;
m = int32(2);
nl = int32(3);
[p, ifail] = s22aa(mode, x, m, nl)
```

```matlab
p =

0
0
2.2500
5.6250
ifail =

0
```