NAG Toolbox for Matlab

s30ba

1 Purpose

s30ba computes the price of a floating-strike lookback option.

2 Syntax

    [p, ifail] = s30ba(calput, sm, s, t, sigma, r, q, 'm', m, 'n', n)

3 Description

s30ba computes the price of a floating-strike lookback call or put option. A call option of this type confers the right to buy the underlying asset at the lowest price, $S_{\text{min}}$, observed during the lifetime of the contract. A put option gives the holder the right to sell the underlying asset at the maximum price, $S_{\text{max}}$, observed during the lifetime of the contract. Thus, at expiry, the payoff for a call option is $S - S_{\text{min}}$, and for a put, $S_{\text{max}} - S$.

For a given minimum value the price of a floating-strike lookback call with underlying asset price, $S$, and time to expiry, $T$, is

$$P_{\text{call}} = S e^{-q T} \Phi(a_1) - S_{\text{min}} e^{-r T} \Phi(a_2) + S e^{-r T} \frac{\sigma^2}{2b} \left[ \left( \frac{S}{S_{\text{min}}} \right)^{-2b/\sigma^2} \Phi \left( -a_1 + \frac{2b}{\sigma} \sqrt{T} \right) - e^{b T} \Phi(-a_1) \right],$$

where $b = r - q \neq 0$. The volatility, $\sigma$, risk-free interest rate, $r$, and annualised dividend yield, $q$, are constants. When $r = q$, the option price is given by

$$P_{\text{call}} = S e^{-q T} \Phi(a_1) - S_{\text{min}} e^{-r T} \Phi(a_2) + S e^{-r T} \frac{\sigma}{2} \left[ \phi(a_1) + a_1 \Phi(a_1) - 1 \right].$$

The corresponding put price is (for $b \neq 0$),

$$P_{\text{put}} = S_{\text{max}} e^{-r T} \Phi(-a_2) - S e^{-q T} \Phi(-a_1) + S e^{-r T} \frac{\sigma^2}{2b} \left[ \left( \frac{S}{S_{\text{max}}} \right)^{-2b/\sigma^2} \Phi \left( a_1 - \frac{2b}{\sigma} \sqrt{T} \right) + e^{b T} \Phi(a_1) \right].$$

When $r = q$,

$$P_{\text{put}} = S_{\text{max}} e^{-r T} \Phi(-a_2) - S e^{-q T} \Phi(-a_1) + S e^{-r T} \sigma \sqrt{T} \left[ \Phi(a_1) + a_1 \Phi(a_1) \right].$$

In the above, $\Phi$ denotes the cumulative Normal distribution function,

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy$$

and

$$a_1 = \ln(S/S_m) + (b+\sigma^2/2)T$$

$$a_2 = a_1 - \sigma \sqrt{T}$$

where $S_m$ is taken to be the minimum price attained by the underlying asset, $S_{\text{min}}$, for a call and the maximum price, $S_{\text{max}}$, for a put.

4 References

Goldman B M, Sosin H B and Gatto M A (1979) Path dependent options: buy at the low, sell at the high
Journal of Finance 34 1111–1127
5 Parameters

5.1 Compulsory Input Parameters

1: calput – string
   Determines whether the option is a call or a put.
   calput = 'C'
   A call. The holder has a right to buy.
   calput = 'P'
   A put. The holder has a right to sell.
   Constraint: calput = 'C' or 'P'.

2: sm(m) – double array
   m, the dimension of the array, must satisfy the constraint m ≥ 1.
   sm(i) must contain \( S_{\min}(i) \), the i\(^{\text{th}}\) minimum observed price of the underlying asset when calput = 'C', or \( S_{\max}(i) \), the maximum observed price when calput = 'P', for \( i = 1, 2, \ldots, m \).
   Constraints:
   \[ \text{sm}(i) \geq z \text{ and } \text{sm}(i) \leq 1/z, \text{ where } z = X02AMF(), \text{ the safe range parameter, for } i = 1, 2, \ldots, m; \]
   if calput = 'C', \( \text{sm}(i) \leq S \), for \( i = 1, 2, \ldots, m \);
   if calput = 'P', \( \text{sm}(i) \geq S \), for \( i = 1, 2, \ldots, m \).

3: s – double scalar
   S, the price of the underlying asset.
   Constraint: \( s \geq z \) and \( s \leq 1/z \), where \( z = X02AMF() \), the safe range parameter.

4: t(n) – double array
   n, the dimension of the array, must satisfy the constraint n ≥ 1.
   t(i) must contain \( T_i \), the i\(^{\text{th}}\) time, in years, to expiry, for \( i = 1, 2, \ldots, n \).
   Constraint: \( t(i) \geq z \), where \( z = X02AMF() \), the safe range parameter, for \( i = 1, 2, \ldots, n \).

5: sigma – double scalar
   \( \sigma \), the volatility of the underlying asset. Note that a rate of 15% should be entered as 0.15.
   Constraint: \( \sigma > 0.0 \).

6: r – double scalar
   \( r \), the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.
   Constraint: \( r \geq 0.0 \).

7: q – double scalar
   \( q \), the annual continuous yield rate. Note that a rate of 8% should be entered as 0.08.
   Constraint: \( q \geq 0.0 \).

5.2 Optional Input Parameters

1: m – int32 scalar
   Default: The dimension of the array sm.
the number of minimum or maximum prices to be used.

*Constraint:* \( m \geq 1 \).

2: \( n \) – int32 scalar

*Default:* The dimension of the array \( t \).

the number of times to expiry to be used.

*Constraint:* \( n \geq 1 \).

5.3 **Input Parameters Omitted from the MATLAB Interface**

`ldp`

5.4 **Output Parameters**

1: \( p(ldp,n) \) – double array

The \( m \times n \) array \( p \) contains the computed option prices.

2: \( ifail \) – int32 scalar

\( ifail = 0 \) unless the function detects an error (see Section 6).

6 **Error Indicators and Warnings**

Errors or warnings detected by the function:

\( ifail = 1 \)

On entry, `calput` \( \neq 'C' \) or `P'`.

\( ifail = 2 \)

On entry, \( m \leq 0 \).

\( ifail = 3 \)

On entry, \( n \leq 0 \).

\( ifail = 4 \)

On entry, \( \text{sm}(i) < z \) or \( \text{sm}(i) > 1/z \), where \( z = X02AMF() \), the safe range parameter,

or \( \text{calput} = 'C' \) and \( \text{sm}(i) > S \),

or \( \text{calput} = 'P' \) and \( \text{sm}(i) < S \).

\( ifail = 5 \)

On entry, \( s < z \) or \( s > 1/z \), where \( z = X02AMF() \), the safe range parameter.

\( ifail = 6 \)

On entry, \( t(i) < z \), where \( z = X02AMF() \), the safe range parameter.

\( ifail = 7 \)

On entry, \( sigma \leq 0.0 \).

\( ifail = 8 \)

On entry, \( r < 0.0 \).
ifail = 9
On entry, q < 0.0.

ifail = 11
On entry, ldp < m.

7 Accuracy
The accuracy of the output is dependent on the accuracy of the cumulative Normal distribution function, Φ. This is evaluated using a rational Chebyshev expansion, chosen so that the maximum relative error in the expansion is of the order of the machine precision (see s15ab and s15ad). An accuracy close to machine precision can generally be expected.

8 Further Comments
None.

9 Example

```
put = 'c';
s = 120;
sigma = 0.3;
r = 0.1;
q = 0.06;
sm = [100.0];
t = [0.5];
[p, ifail] = s30ba(put, sm, s, t, sigma, r, q);

fprintf('Floating-strike Lookback
 European Call :
');
fprintf(' Spot = %9.4f
', s);
fprintf(' Volatility = %9.4f
', sigma);
fprintf(' Rate = %9.4f
', r);
fprintf(' Dividend = %9.4f

', q);

fprintf(' Strike Expiry Option Price
');
for i=1:1
  for j=1:1
    fprintf('%9.4f %9.4f %9.4f
', sm(i), t(j), p(i,j));
  end
end
```

Floating-strike Lookback
European Call :
Spot = 120.0000
Volatility = 0.3000
Rate = 0.1000
Dividend = 0.0600

Strike Expiry Option Price
100.0000 0.5000 25.3534