NAG Toolbox for Matlab

s30bb

1 Purpose

s30bb computes the price of a floating-strike lookback option together with its sensitivities (Greeks).

2 Syntax

\[ [p, \text{delta}, \text{gamma}, \text{vega}, \text{theta}, \text{rho}, \text{crho}, \text{vanna}, \text{charm}, \text{speed}, \text{colour}, \text{zomma}, \text{vomma}, \text{ifail}] = s30bb(\text{calput}, \text{sm}, \text{s}, \text{t}, \text{sigma}, \text{r}, \text{q}, 'm', \text{m}, 'n', \text{n}) \]

3 Description

s30bb computes the price of a floating-strike lookback call or put option, together with the Greeks or sensitivities, which are the partial derivatives of the option price with respect to certain of the other input parameters. A call option of this type confers the right to buy the underlying asset at the lowest price, \( S_{\text{min}} \), observed during the lifetime of the contract. A put option gives the holder the right to sell the underlying asset at the maximum price, \( S_{\text{max}} \), observed during the lifetime of the contract. Thus, at expiry, the payoff for a call option is \( \frac{S}{C_0} \cdot S_{\text{min}} \), and for a put, \( \frac{S_{\text{max}}}{C_0} \).

For a given minimum value the price of a floating-strike lookback call with underlying asset price, \( S \), and time to expiry, \( T \), is

\[
P_{\text{call}} = S e^{-\sigma^2 T} \Phi(a_1) - S_{\text{min}} e^{-\sigma^2 T} \Phi(a_2) + S e^{-\sigma^2 T} \left\{ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a_1} e^{-y^2/2} dy \right\},
\]

where \( b = r - q \neq 0 \). The volatility, \( \sigma \), risk-free interest rate, \( r \), and annualised dividend yield, \( q \), are constants.

The corresponding put price is

\[
P_{\text{put}} = S_{\text{max}} e^{-\sigma^2 T} \Phi(-a_2) - S e^{-\sigma^2 T} \Phi(-a_1) + S e^{-\sigma^2 T} \left\{ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a_1} e^{-y^2/2} dy \right\}.
\]

In the above, \( \Phi \) denotes the cumulative Normal distribution function,

\[
\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy
\]

and

\[
a_1 = \frac{\ln(S_{\text{min}}) + (b+\sigma^2/2)T}{\sigma \sqrt{T}}
\]

\[
a_2 = a_1 - \sigma \sqrt{T}
\]

where \( S_{\text{min}} \) is taken to be the minimum price attained by the underlying asset, \( S_{\text{min}} \), for a call and the maximum price, \( S_{\text{max}} \), for a put.

4 References

Goldman B M, Sosin H B and Gatto M A (1979) Path dependent options: buy at the low, sell at the high Journal of Finance 34 1111–1127
5 Parameters

5.1 Compulsory Input Parameters

1: calput – string
   Determines whether the option is a call or a put.
   \[\text{calput} = 'C'\]
   A call. The holder has a right to buy.
   \[\text{calput} = 'P'\]
   A put. The holder has a right to sell.
   \[\text{Constraint: calput} = 'C' \text{ or } 'P'.\]

2: \(sm(m)\) – double array
   \(m\), the dimension of the array, must satisfy the constraint \(m \geq 1\).
   \[sm(i)\] must contain \(S_{\text{min}}(i)\), the \(i\)th minimum observed price of the underlying asset when \(\text{calput} = 'C'\), or \(S_{\text{max}}(i)\), the maximum observed price when \(\text{calput} = 'P'\), for \(i = 1, 2, \ldots, m\).
   \[\text{Constraints:}\]
   \[sm(i) \geq z \text{ and } sm(i) \leq 1/z, \text{ where } z = X02AMF(), \text{ the safe range parameter, for } i = 1, 2, \ldots, m;\]
   if \(\text{calput} = 'C'\), \(sm(i) \leq S\), for \(i = 1, 2, \ldots, m;\)
   if \(\text{calput} = 'P'\), \(sm(i) \geq S\), for \(i = 1, 2, \ldots, m.\)

3: \(s\) – double scalar
   \(S\), the price of the underlying asset.
   \[\text{Constraint: } s \geq z \text{ and } s \leq 1/z, \text{ where } z = X02AMF(), \text{ the safe range parameter.}\]

4: \(t(n)\) – double array
   \(n\), the dimension of the array, must satisfy the constraint \(n \geq 1\).
   \(t(i)\) must contain \(T_i\), the \(i\)th time, in years, to expiry, for \(i = 1, 2, \ldots, n\).
   \[\text{Constraint: } t(i) \geq z, \text{ where } z = X02AMF(), \text{ the safe range parameter, for } i = 1, 2, \ldots, n.\]

5: \(sigma\) – double scalar
   \(\sigma\), the volatility of the underlying asset. Note that a rate of 15% should be entered as 0.15.
   \[\text{Constraint: } sigma > 0.0.\]

6: \(r\) – double scalar
   The annual risk-free interest rate, \(r\), continuously compounded. Note that a rate of 5% should be entered as 0.05.
   \[\text{Constraint: } r \geq 0.0 \text{ and } r \neq q.\]

7: \(q\) – double scalar
   The annual continuous yield rate. Note that a rate of 8% should be entered as 0.08.
   \[\text{Constraint: } q \geq 0.0 \text{ and } q \neq r.\]

5.2 Optional Input Parameters

1: \(m\) – int32 scalar
   Default: The dimension of the array \(sm\).
the number of minimum or maximum prices to be used.

Constraint: \( m \geq 1 \).

2: \( n \) – int32 scalar

Default: The dimension of the array \( t \).

the number of times to expiry to be used.

Constraint: \( n \geq 1 \).

5.3 Input Parameters Omitted from the MATLAB Interface

ldp

5.4 Output Parameters

1: \( p(ldp,n) \) – double array

The \( m \times n \) array \( p \) contains the computed option prices.

2: \( \text{delta}(ldp,n) \) – double array

The \( m \times n \) array \( \text{delta} \) contains the sensitivity, \( \frac{\partial P}{\partial S} \), of the option price to change in the price of the underlying asset.

3: \( \text{gamma}(ldp,n) \) – double array

The \( m \times n \) array \( \text{gamma} \) contains the sensitivity, \( \frac{\partial^2 P}{\partial S^2} \), of \( \text{delta} \) to change in the price of the underlying asset.

4: \( \text{vega}(ldp,n) \) – double array

The \( m \times n \) array \( \text{vega} \) contains the sensitivity, \( \frac{\partial P}{\partial \sigma} \), of the option price to change in the volatility of the underlying asset.

5: \( \text{theta}(ldp,n) \) – double array

The \( m \times n \) array \( \text{theta} \) contains the sensitivity, \( -\frac{\partial P}{\partial T} \), of the option price to change in the time to expiry of the option.

6: \( \text{rho}(ldp,n) \) – double array

The \( m \times n \) array \( \text{rho} \) contains the sensitivity, \( \frac{\partial P}{\partial r} \), of the option price to change in the annual risk-free interest rate.

7: \( \text{crho}(ldp,n) \) – double array

The \( m \times n \) array \( \text{crho} \) containing the sensitivity, \( \frac{\partial^2 P}{\partial S \partial b} \), of the option price to change in the annual cost of carry rate, \( b \), where \( b = r - q \).

8: \( \text{vanna}(ldp,n) \) – double array

The \( m \times n \) array \( \text{vanna} \) contains the sensitivity, \( \frac{\partial^2 P}{\partial S \partial \sigma} \), of \( \text{vega} \) to change in the price of the underlying asset or, equivalently, the sensitivity of \( \text{delta} \) to change in the volatility of the asset price.

9: \( \text{charm}(ldp,n) \) – double array

The \( m \times n \) array \( \text{charm} \) contains the sensitivity, \( -\frac{\partial^2 P}{\partial S \partial T} \), of \( \text{delta} \) to change in the time to expiry of the option.
10: speed(ldp,n) – double array
The \( m \times n \) array speed contains the sensitivity, \( \frac{\partial p}{\partial s} \), of gamma to change in the price of the underlying asset.

11: colour(ldp,n) – double array
The \( m \times n \) array colour contains the sensitivity, \( -\frac{\partial p}{\partial S} \), of gamma to change in the time to expiry of the option.

12: zomma(ldp,n) – double array
The \( m \times n \) array zomma contains the sensitivity, \( \frac{\partial p}{\partial \sigma} \), of gamma to change in the volatility of the underlying asset.

13: vomma(ldp,n) – double array
The \( m \times n \) array vomma contains the sensitivity, \( \frac{\partial^2 p}{\partial \sigma^2} \), of vega to change in the volatility of the underlying asset.

14: ifail – int32 scalar
ifail = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings
Errors or warnings detected by the function:

ifail = 1
On entry, calput \( \neq \) 'C' or 'P'.

ifail = 2
On entry, m \( \leq 0 \).

ifail = 3
On entry, n \( \leq 0 \).

ifail = 4
On entry, \( sm(i) < z \) or \( sm(i) > 1/z \), where \( z = X02AMF() \), the safe range parameter,
or calput = 'C' and \( sm(i) > S \),
or calput = 'P' and \( sm(i) < S \).

ifail = 5
On entry, s \( < z \) or \( s > 1/z \), where \( z = X02AMF() \), the safe range parameter.

ifail = 6
On entry, t(i) \( < z \), where \( z = X02AMF() \), the safe range parameter.

ifail = 7
On entry, sigma \( \leq 0.0 \).

ifail = 8
On entry, r \( < 0.0 \).
ifail = 9
On entry, \( q < 0.0 \).

ifail = 11
On entry, \( ldp < m \).

ifail = 12
On entry, \( r = q \).

7 Accuracy
The accuracy of the output is dependent on the accuracy of the cumulative Normal distribution function, \( \Phi \). This is evaluated using a rational Chebyshev expansion, chosen so that the maximum relative error in the expansion is of the order of the machine precision (see s15ab and s15ad). An accuracy close to machine precision can generally be expected.

8 Further Comments
None.

9 Example

```matlab
put = 'p';
s = 87;
sigma = 0.3;
r = 0.06;
q = 0.04;
sm = [100.0];
t = [0.5];

[p, delta, gamma, vega, theta, rho, crho, vanna, charm, speed, colour, ...
 zomma, vomma, ifail] = s30bb(put, sm, s, t, sigma, r, q);
fprintf('
Floating-Strike Lookback
 European Put :
');
fprintf(' Spot = %9.4f
', s);
fprintf(' Volatility = %9.4f
', sigma);
fprintf(' Rate = %9.4f
', r);
fprintf(' Dividend = %9.4f

', q);
fprintf(' Time to Expiry : %8.4f
', t(1));
fprintf(' S-Max/Min Price Delta Gamma Vega Theta Rho
 CRho
');
fprintf(' %8.4f %8.4f %8.4f %8.4f %8.4f %8.4f %8.4f %8.4f
', sm(1), ...
 p(1,1), delta(1,1), gamma(1,1), vega(1,1), theta(1,1), rho(1,1), ...
 crho(1,1));
fprintf(' S-Max/Min Price Vanna Charm Speed Colour Zomma
 Vomma
');
fprintf(' %8.4f %8.4f %8.4f %8.4f %8.4f %8.4f %8.4f %8.4f
', sm(1), ...
 p(1,1), vanna(1,1), charm(1,1), speed(1,1), colour(1,1), ...
 zomma(1,1), vomma(1,1));
```

Floating-Strike Lookback
European Put :
Spot = 87.0000
Volatility = 0.3000
Rate = 0.0600
| Dividend | 0.0400 |
| Time to Expiry | 0.5000 |
| S-Max/Min Price | Delta | Gamma | Vega | Theta | Rho | CRho |
| 100.0000 | 18.3530 | -0.3560 | 0.0391 | 45.5353 | -11.6139 | -32.8139 | -23.6374 |
| S-Max/Min Price | Vanna | Charm | Speed | Colour | Zomma | Vomma |
| 100.0000 | 18.3530 | 1.9141 | -0.6199 | 0.0007 | 0.0221 | -0.0648 | 76.1292 |