1 Purpose

s30ca computes the price of a binary or digital cash-or-nothing option.

2 Syntax

\[ [p, ifail] = \text{s30ca}(\text{calput}, x, s, k, t, \sigma, r, q, 'm', m, 'n', n) \]

3 Description

s30ca computes the price of a binary or digital cash-or-nothing option which pays a fixed amount, \( K \), at expiration if the option is in-the-money (see Section 2.4 in the S Chapter Introduction). For a strike price, \( X \), underlying asset price, \( S \), and time to expiry, \( T \), the payoff is therefore \( K \), if \( S > X \) for a call or \( S < X \) for a put. Nothing is paid out when this condition is not met.

The price of a call with volatility, \( \sigma \), risk-free interest rate, \( r \), and annualised dividend yield, \( q \), is

\[ P_{\text{call}} = K e^{-rT} \Phi(d_2) \]

and for a put,

\[ P_{\text{put}} = K e^{-rT} \Phi(-d_2) \]

where \( \Phi \) is the cumulative Normal distribution function,

\[ \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} (-y^2/2) dy, \]

and

\[ d_2 = \ln(S/X) + (r - q - \sigma^2/2)T \]

\[ \sigma\sqrt{T} \]

4 References

Reiner E and Rubinstein M (1991) Unscrambling the binary code Risk 4

5 Parameters

5.1 Compulsory Input Parameters

1: \ calput – string

Determines whether the option is a call or a put.

\ calput = 'C'

A call. The holder has a right to buy.

\ calput = 'P'

A put. The holder has a right to sell.

Constraint: \ calput = 'C' or 'P'.

2: \ x(m) – double array

\ m, the dimension of the array, must satisfy the constraint \( m \geq 1 \).
\( x(i) \) must contain \( X_i \), the \( i \)th strike price, for \( i = 1, 2, \ldots, m \).

**Constraint:** \( x(i) \geq z \) and \( x(i) \leq 1/z \), where \( z = X02AMF() \), the safe range parameter, for \( i = 1, 2, \ldots, m \).

3: \textbf{s – double scalar}

\( S \), the price of the underlying asset.

**Constraint:** \( s \geq z \) and \( s \leq 1/z \), where \( z = X02AMF() \), the safe range parameter.

4: \textbf{k – double scalar}

The amount, \( K \), to be paid at expiration if the option is in-the-money, i.e., if \( s > x(i) \) when \texttt{calput = 'C',} or if \( s < x(i) \) when \texttt{calput = 'P',} for \( i = 1, 2, \ldots, m \).

**Constraint:** \( k \geq 0.0 \).

5: \textbf{t(n) – double array}

\( n \), the dimension of the array, must satisfy the constraint \( n \geq 1 \).

\( t(i) \) must contain \( T_i \), the \( i \)th time, in years, to expiry, for \( i = 1, 2, \ldots, n \).

**Constraint:** \( t(i) \geq z \), where \( z = X02AMF() \), the safe range parameter, for \( i = 1, 2, \ldots, n \).

6: \textbf{sigma – double scalar}

\( \sigma \), the volatility of the underlying asset. Note that a rate of 15\% should be entered as 0.15.

**Constraint:** \( \sigma > 0.0 \).

7: \textbf{r – double scalar}

\( r \), the annual risk-free interest rate, continuously compounded. Note that a rate of 5\% should be entered as 0.05.

**Constraint:** \( r \geq 0.0 \).

8: \textbf{q – double scalar}

\( q \), the annual continuous yield rate. Note that a rate of 8\% should be entered as 0.08.

**Constraint:** \( q \geq 0.0 \).

5.2 \textbf{Optional Input Parameters}

1: \textbf{m – int32 scalar}

*Default:* The dimension of the array \( x \). the number of strike prices to be used.

**Constraint:** \( m \geq 1 \).

2: \textbf{n – int32 scalar}

*Default:* The dimension of the array \( t \). the number of times to expiry to be used.

**Constraint:** \( n \geq 1 \).

5.3 \textbf{Input Parameters Omitted from the MATLAB Interface}

\texttt{ldp}
5.4 Output Parameters
1: \( p(ldp,n) \) – double array
   The \( m \times n \) array \( p \) contains the computed option prices.
2: ifail – int32 scalar
   ifail = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings
Errors or warnings detected by the function:

ifail = 1
   On entry, calput \( \neq 'C' \) or 'P'.

ifail = 2
   On entry, \( m \leq 0 \).

ifail = 3
   On entry, \( n \leq 0 \).

ifail = 4
   On entry, \( x(i) < z \) or \( x(i) > 1/z \), where \( z = X02AMF() \), the safe range parameter.

ifail = 5
   On entry, \( s < z \) or \( s > 1/z \), where \( z = X02AMF() \), the safe range parameter.

ifail = 6
   On entry, \( k < 0.0 \).

ifail = 7
   On entry, \( t(i) < z \), where \( z = X02AMF() \), the safe range parameter.

ifail = 8
   On entry, \( \text{sigma} \leq 0.0 \).

ifail = 9
   On entry, \( r < 0.0 \).

ifail = 10
   On entry, \( q < 0.0 \).

ifail = 12
   On entry, \( ldp < m \).

7 Accuracy
The accuracy of the output is dependent on the accuracy of the cumulative Normal distribution function, \( \Phi \). This is evaluated using a rational Chebyshev expansion, chosen so that the maximum relative error in the expansion is of the order of the machine precision (see s15ab and s15ad). An accuracy close to machine precision can generally be expected.
8 Further Comments
None.

9 Example

```matlab
put = 'p';
s = 100.0;
k = 10.0;
sigma = 0.35;
r = 0.06;
q = 0.0;
x = [80.0];
t = [0.75];
[p, ifail] = s30ca(put, x, s, k, t, sigma, r, q);

fprintf('
Binary (Digital): Cash-or-Nothing
European Put :
');
fprintf(' Spot = %9.4f
', s);
fprintf(' Payout = %9.4f
', k);
fprintf(' Volatility = %9.4f
', sigma);
fprintf(' Rate = %9.4f
', r);
fprintf(' Dividend = %9.4f

', q);

fprintf(' Strike Expiry Option Price
');
for i=1:1
    for j=1:1
        fprintf('%9.4f %9.4f %9.4f
', x(i), t(j), p(i,j));
    end
end
```

Binary (Digital): Cash-or-Nothing
European Put :
Spot = 100.0000
Payout = 10.0000
Volatility = 0.3500
Rate = 0.0600
Dividend = 0.0000

Strike Expiry Option Price
80.0000 0.7500 2.2155