NAG Toolbox for Matlab

s30fa

1 Purpose
s30fa computes the price of a standard barrier option.

2 Syntax
\[ [p, ifail] = s30fa(calput, type, x, s, h, k, t, sigma, r, q, 'm', m, 'n', n) \]

3 Description
s30fa computes the price of a standard barrier option, where the exercise, for a given strike price, \( X \), depends on the underlying asset price, \( S \), reaching or crossing a specified barrier level, \( H \). Barrier options of type \( \text{In} \) only become active (are knocked in) if the underlying asset price attains the pre-determined barrier level during the lifetime of the contract. Those of type \( \text{Out} \) start active and are knocked out if the underlying asset price attains the barrier level during the lifetime of the contract. A cash rebate, \( K \), may be paid if the option is inactive at expiration. The option may also be described as \( \text{Up} \) (the underlying price starts below the barrier level) or \( \text{Down} \) (the underlying price starts above the barrier level). This gives the following options which can be specified as put or call contracts.

**Down-and-In**: the option starts inactive with the underlying asset price above the barrier level. It is knocked in if the underlying price moves down to hit the barrier level before expiration.

**Down-and-Out**: the option starts active with the underlying asset price above the barrier level. It is knocked out if the underlying price moves down to hit the barrier level before expiration.

**Up-and-In**: the option starts inactive with the underlying asset price below the barrier level. It is knocked in if the underlying price moves up to hit the barrier level before expiration.

**Up-and-Out**: the option starts active with the underlying asset price below the barrier level. It is knocked out if the underlying price moves up to hit the barrier level before expiration.

The payoff is \( \max(S - X, 0) \) for a call or \( \max(X - S, 0) \) for a put, if the option is active at expiration, otherwise it may pay a pre-specified cash rebate, \( K \). Following Haug (2007), the prices of the various standard barrier options can be written as shown below. The volatility, \( \sigma \), risk-free interest rate, \( r \), and annualised dividend yield, \( q \), are constants. The integer parameters, \( j \) and \( k \), take the values \( \pm 1 \), depending on the type of barrier.

\[
A = j S e^{-\tau T} \Phi(j x_1) - j S e^{-\tau T} \Phi\left(j \left[ x_1 - \sigma \sqrt{T} \right]\right)
\]
\[
B = j S e^{-\tau T} \Phi(j x_2) - j S e^{-\tau T} \Phi\left(j \left[ x_2 - \sigma \sqrt{T} \right]\right)
\]
\[
C = j S e^{-\tau T} \left(\frac{H}{\sigma}\right)^{2|\mu + 1|} \Phi(k y_1) - j S e^{-\tau T} \left(\frac{H}{\sigma}\right)^{2|\mu|} \Phi\left(k \left[ y_1 - \sigma \sqrt{T} \right]\right)
\]
\[
D = j S e^{-\tau T} \left(\frac{H}{\sigma}\right)^{2|\mu + 1|} \Phi(k y_2) - j S e^{-\tau T} \left(\frac{H}{\sigma}\right)^{2|\mu|} \Phi\left(k \left[ y_2 - \sigma \sqrt{T} \right]\right)
\]
\[
E = Ke^{-\tau T} \left\{ \Phi\left(k \left[ x_2 - \sigma \sqrt{T} \right]\right) - \left(\frac{H}{\sigma}\right)^{2|\mu|} \Phi\left(k \left[ y_2 - \sigma \sqrt{T} \right]\right) \right\}
\]
\[
F = K \left\{ \left(\frac{H}{\sigma}\right)^{n+\lambda} \Phi(k z) + \left(\frac{H}{\sigma}\right)^{n-\lambda} \Phi\left(k \left[ z - \sigma \sqrt{T} \right]\right) \right\}
\]

with
\[ x_1 = \frac{\ln(S/X)}{\sigma \sqrt{T}} + (1 + \mu)\sigma \sqrt{T} \]
\[ x_2 = \frac{\ln(S/H)}{\sigma \sqrt{T}} + (1 + \mu)\sigma \sqrt{T} \]
\[ y_1 = \frac{\ln(H/(SX))}{\sigma \sqrt{T}} + (1 + \mu)\sigma \sqrt{T} \]
\[ y_2 = \frac{\ln(H/S)}{\sigma \sqrt{T}} + (1 + \mu)\sigma \sqrt{T} \]
\[ z = \frac{\ln(H/S)}{\sigma \sqrt{T}} + \lambda \sigma \sqrt{T} \]
\[ \mu = \frac{r - \frac{\sigma^2}{2}}{\sigma} \]
\[ \lambda = \sqrt{\mu^2 + \frac{2r}{\sigma^2}} \]

**Down-and-In (S > H):**
When \( X \geq H \), with \( j = k = 1 \),
\[ P_{\text{call}} = C + E \]
and with \( j = -1, k = 1 \)
\[ P_{\text{put}} = B - C + D + E \]

When \( X < H \), with \( j = k = 1 \)
\[ P_{\text{call}} = A - B + D + E \]
and with \( j = -1, k = 1 \)
\[ P_{\text{put}} = A + E \]

**Down-and-Out (S > H):**
When \( X \geq H \), with \( j = k = 1 \),
\[ P_{\text{call}} = A - C + F \]
and with \( j = -1, k = 1 \)
\[ P_{\text{put}} = A - B + C - D + F \]

When \( X < H \), with \( j = k = 1 \),
\[ P_{\text{call}} = B - D + F \]
and with \( j = -1, k = 1 \)
\[ P_{\text{put}} = F \]

**Up-and-In (S < H):**
When \( X \geq H \), with \( j = 1, k = -1 \),
\[ P_{\text{call}} = A + E \]
and with \( j = k = -1 \),
\[ P_{\text{put}} = A - B + D + E \]

When \( X < H \), with \( j = 1, k = -1 \),
\[ P_{\text{call}} = B - C + D + E \]
and with \( j = k = -1 \),
\[ P_{\text{put}} = C + E \]
Up-and-Out ($S < H$):
When $X \geq H$, with $j = 1$, $k = -1$,
$$ P_{\text{call}} = F $$
and with $j = k = -1$,
$$ P_{\text{put}} = B - D + F $$
When $X < H$, with $j = 1$, $k = -1$,
$$ P_{\text{call}} = A - B + C - D + F $$
and with $j = k = -1$,
$$ P_{\text{put}} = A - C + F $$

4 References

5 Parameters
5.1 Compulsory Input Parameters
1: calput – string
   Determines whether the option is a call or a put.
   calput = 'C'
      A call. The holder has a right to buy.
   calput = 'P'
      A put. The holder has a right to sell.
   Constraint: calput = 'C' or 'P'.

2: type – string
   Indicates the barrier type as In or Out and its relation to the price of the underlying asset as Up or Down.
   type = 'DI'
      Down-and-In.
   type = 'DO'
      Down-and-Out.
   type = 'UI'
      Up-and-In.
   type = 'UO'
      Up-and-Out.
   Constraint: type = 'DI', 'DO', 'UI' or 'UO'.

3: x(m) – double array
   m, the dimension of the array, must satisfy the constraint $m \geq 1$.
   x(i) must contain $X_i$, the $i$th strike price, for $i = 1, 2, \ldots, m$.
   Constraint: $x(i) \geq z$ and $x(i) \leq 1/z$, where $z = X02AMF()$, the safe range parameter, for $i = 1, 2, \ldots, m$. 

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4: \( s \) – double scalar

\( S \), the price of the underlying asset.

*Constraint*: \( s \geq z \) and \( s \leq 1/z \), where \( z = X02AMF() \), the safe range parameter.

5: \( h \) – double scalar

The barrier price.

*Constraint*: \( h \geq z \) and \( h \leq 1/z \), where \( z = X02AMF() \), the safe range parameter.

6: \( k \) – double scalar

The value of a possible cash rebate to be paid if the option has not been knocked in (or out) before expiration.

*Constraint*: \( k \geq 0 \).

7: \( t(n) \) – double array

\( n \), the dimension of the array, must satisfy the constraint \( n \geq 1 \).

\( t(i) \) must contain \( T_i \), the \( i \)th time, in years, to expiry, for \( i = 1, 2, \ldots, n \).

*Constraint*: \( t(i) \geq z \), where \( z = X02AMF() \), the safe range parameter, for \( i = 1, 2, \ldots, n \).

8: \( \sigma \) – double scalar

\( \sigma \), the volatility of the underlying asset. Note that a rate of 15% should be entered as 0.15.

*Constraint*: \( \sigma > 0 \).

9: \( r \) – double scalar

\( r \), the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.

*Constraint*: \( r \geq 0.0 \).

10: \( q \) – double scalar

\( q \), the annual continuous yield rate. Note that a rate of 8% should be entered as 0.08.

*Constraint*: \( q \geq 0.0 \).

5.2 Optional Input Parameters

1: \( m \) – int32 scalar

*Default*: The dimension of the array \( x \). the number of strike prices to be used.

*Constraint*: \( m \geq 1 \).

2: \( n \) – int32 scalar

*Default*: The dimension of the array \( t \). the number of times to expiry to be used.

*Constraint*: \( n \geq 1 \).

5.3 Input Parameters Omitted from the MATLAB Interface

ldp
5.4 Output Parameters

1: \( p(ldp,n) \) – double array
   The \( m \times n \) array \( p \) contains the computed option prices.

2: ifail – int32 scalar
   ifail = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1
   On entry, calput \( \neq 'C' \) or 'P'.

ifail = 2
   On entry, type \( \neq 'DI', 'DO', 'UI' \) or 'UO'.

ifail = 3
   On entry, \( m \leq 0. \)

ifail = 4
   On entry, \( n \leq 0. \)

ifail = 5
   On entry, \( x(i) < z \) or \( x(i) > 1/z \), where \( z = X02AMF() \), the safe range parameter.

ifail = 6
   On entry, \( s < z \) or \( s > 1/z \), where \( z = X02AMF() \), the safe range parameter.

ifail = 7
   On entry, \( h < z \) or \( h > 1/z \), where \( z = X02AMF() \), the safe range parameter.

ifail = 8
   On entry, \( k < 0. \)

ifail = 9
   On entry, \( t(i) < z \), where \( z = X02AMF() \), the safe range parameter.

ifail = 10
   On entry, sigma \( \leq 0.0. \)

ifail = 11
   On entry, \( r < 0.0. \)

ifail = 12
   On entry, \( q < 0.0. \)

ifail = 15
   \( s \) and \( h \) are not consistent with type.
ifail = 14
On entry, ldp < m.

7 Accuracy
The accuracy of the output is dependent on the accuracy of the cumulative Normal distribution function, \( \Phi \). This is evaluated using a rational Chebyshev expansion, chosen so that the maximum relative error in the expansion is of the order of the **machine precision** (see s15ab and s15ad). An accuracy close to **machine precision** can generally be expected.

8 Further Comments
None.

9 Example

```matlab
put = 'P';
type = 'DI';
s = 100.0;
h = 95.0;
k = 3.0;
sigma = 0.3;
x = 0.08;
q = 0.04;
x = [100.0];
t = [0.5];

[p, ifail] = s30fa(put, type, x, s, h, k, t, sigma, r, q);
```

```matlab
fprintf('
Standard Barrier Option
Put :
');
fprintf(' Spot = %9.4f
', s);
fprintf(' Barrier = %9.4f
', h);
fprintf(' Rebate = %9.4f
', k);
fprintf(' Volatility = %9.4f
', sigma);
fprintf(' Rate = %9.4f
', r);
fprintf(' Dividend = %9.4f

', q);
fprintf(' Strike Expiry Option Price
');
for i=1:1
for j=1:1
fprintf('%9.4f %9.4f %9.4f
', x(i), t(j), p(i,j));
end
end
```

```
Standard Barrier Option
Put :
Spot = 100.0000
Barrier = 95.0000
Rebate = 3.0000
Volatility = 0.3000
Rate = 0.0800
Dividend = 0.0400

Strike Expiry Option Price
100.0000 0.5000 7.7988
```