NAG Toolbox for Matlab

s30qc

1 Purpose

s30qc computes the Bjerksund and Stensland (2002) approximation to the price of an American option.

2 Syntax

\[ [p, ifail] = s30qc(calput, x, s, t, sigma, r, q, 'm', m, 'n', n) \]

3 Description

s30qc computes the price of an American option using the closed form approximation of Bjerksund and Stensland (2002). The time to maturity, \( T \), is divided into two periods, each with a flat early exercise boundary, by choosing a time \( t = \frac{1}{2}(\sqrt{5} - 1)T \). The two boundary values are defined as

\[
\tilde{x} = \tilde{X}(t), \quad \tilde{X} = \tilde{X}(T)
\]

with

\[
\tilde{X}(t) = B_0 + (B_\infty - B_0)(1 - \exp\{h(\tau)\}),
\]

where

\[
h(\tau) = -(br + 2\sigma\sqrt{\tau})\left(\frac{X^2}{(B_\infty - B_0)B_0}\right),
\]

\[
B_\infty \equiv \frac{\beta}{\beta - 1}X, \quad B_0 \equiv \max\{X, \left(\frac{r}{r-b}\right)X\},
\]

\[
\beta = \left(1 - \frac{b}{\sigma^2}\right) + \sqrt{\left(\frac{b}{\sigma^2} - \frac{1}{2}\right)^2 + 2\frac{r}{\sigma^2}}
\]

with \( b = r - q \), the cost of carry, where \( r \) is the risk-free interest rate and \( q \) is the annual dividend rate. Here \( X \) is the strike price and \( \sigma \) is the annual volatility.

The price of an American call option is approximated as

\[
P_{\text{call}} = \alpha(\tilde{X})S^\beta - \alpha(\tilde{X})\phi(S, t|\beta, \tilde{X}, \tilde{X}) + \phi(S, t|1, \tilde{X}, \tilde{X}) - \phi(S, t|1, \tilde{x}, \tilde{X}) - X\phi(S, t|0, \tilde{X}, \tilde{X}) + X\phi(S, t|0, \tilde{x}, \tilde{X}) + \alpha(\tilde{x})\phi(S, t|\beta, \tilde{x}, \tilde{X}) - \alpha(\tilde{x})\psi(S, T|\beta, \tilde{x}, \tilde{X}, \tilde{x}, t) + \psi(S, T|1, \tilde{x}, \tilde{X}, \tilde{x}, t) - \psi(S, T|1, X, \tilde{x}, \tilde{x}, t) - X\psi(S, T|0, \tilde{x}, \tilde{X}, \tilde{x}, t) + X\psi(S, T|0, X, \tilde{X}, \tilde{x}, t),
\]

where \( \alpha, \phi \) and \( \psi \) are as defined in Bjerksund and Stensland (2002).

The price of a put option is obtained by the put-call transformation,

\[
P_{\text{put}}(X, S, T, \sigma, r, q) = P_{\text{call}}(S, X, T, \sigma, q, r).
\]

4 References


5 Parameters

5.1 Compulsory Input Parameters
1: calput – string
   Determines whether the option is a call or a put.
   \[ \text{calput} = 'C' \]
   A call. The holder has a right to buy.
   \[ \text{calput} = 'P' \]
   A put. The holder has a right to sell.
   Constraint: \( \text{calput} = 'C' \) or \( 'P' \).

2: \( x(m) \) – double array
   \( m \), the dimension of the array, must satisfy the constraint \( m \geq 1 \).
   \( x(i) \) must contain \( X_i \), the \( i \)th strike price, for \( i = 1, 2, \ldots, m \).
   Constraint: \( x(i) \geq z \) and \( x(i) \leq 1/z \), where \( z = \text{X02AMF()} \), the safe range parameter, for \( i = 1, 2, \ldots, m \).

3: s – double scalar
   \( S \), the price of the underlying asset.
   Constraint: \( s \geq z \) and \( s \leq 1/z \), where \( z = \text{X02AMF()} \), the safe range parameter.

4: \( t(n) \) – double array
   \( n \), the dimension of the array, must satisfy the constraint \( n \geq 1 \).
   \( t(i) \) must contain \( T_i \), the \( i \)th time, in years, to expiry, for \( i = 1, 2, \ldots, n \).
   Constraint: \( t(i) \geq z \), where \( z = \text{X02AMF()} \), the safe range parameter, for \( i = 1, 2, \ldots, n \).

5: sigma – double scalar
   \( \sigma \), the volatility of the underlying asset. Note that a rate of 15% should be entered as 0.15.
   Constraint: \( \sigma > 0.0 \).

6: r – double scalar
   \( r \), the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.
   Constraint: \( r \geq 0.0 \).

7: q – double scalar
   \( q \), the annual continuous yield rate. Note that a rate of 8% should be entered as 0.08.
   Constraint: \( q \geq 0.0 \).

5.2 Optional Input Parameters
1: m – int32 scalar
   Default: The dimension of the array \( x \).
   the number of strike prices to be used.
   Constraint: \( m \geq 1 \).
2: \textbf{n} – \texttt{int32 scalar}

*Default:* The dimension of the array \texttt{t}.

the number of times to expiry to be used.

*Constraint:* \texttt{n} \geq 1.

5.3 Input Parameters Omitted from the MATLAB Interface

\texttt{ldp}

5.4 Output Parameters

1: \texttt{p(ldp,n)} – double array

The \( m \times n \) array \texttt{p} contains the computed option prices.

2: \texttt{ifail} – \texttt{int32 scalar}

\texttt{ifail} = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

\texttt{ifail} = 1

On entry, \texttt{calput} \neq 'C' or 'P'.

\texttt{ifail} = 2

On entry, \texttt{m} \leq 0.

\texttt{ifail} = 3

On entry, \texttt{n} \leq 0.

\texttt{ifail} = 4

On entry, \texttt{x(i)} < z or \texttt{x(i)} > 1/z, where \( z = X02AMF() \), the safe range parameter.

\texttt{ifail} = 5

On entry, \texttt{s} < z or \texttt{s} > 1/z, where \( z = X02AMF() \), the safe range parameter.

\texttt{ifail} = 6

On entry, \texttt{t(i)} < z, where \( z = X02AMF() \), the safe range parameter.

\texttt{ifail} = 7

On entry, \texttt{sigma} \leq 0.0.

\texttt{ifail} = 8

On entry, \texttt{r} < 0.0.

\texttt{ifail} = 9

On entry, \texttt{q} < 0.0.

\texttt{ifail} = 11

On entry, \texttt{ldp} < \texttt{m}. 

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ifail = 14

On entry, $\beta \geq \frac{1}{2}$, where $z = \text{X02AMF}()$, the safe range parameter (see Section 3).

ifail = 15

Internal memory allocation failed.

7 Accuracy

The accuracy of the output will be bounded by the accuracy of the cumulative bivariate Normal distribution function. The algorithm of Genz (2004) is used, as described in the document for g01ha, giving a maximum absolute error of less than $5 \times 10^{-16}$. The univariate cumulative Normal distribution function also forms part of the evaluation (see s15ab and s15ad).

8 Further Comments

None.

9 Example

```matlab
put = 'c';
s = 110.0;
sigma = 0.2;
r = 0.08;
q = 0.12;
x = [100.0];
t = [0.25];
[p, ifail] = s30qc(put, x, s, t, sigma, r, q);

fprintf('
American Call :
');
fprintf(' Spot = %9.4f
', s);
fprintf(' Volatility = %9.4f
', sigma);
fprintf(' Rate = %9.4f
', r);
fprintf(' Dividend = %9.4f

', q);
fprintf(' Strike Expiry Option Price
');
for i=1:1
    for j=1:1
        fprintf('%9.4f %9.4f %9.4f
', x(i), t(j), p(i,j));
    end
end
```

American Call :
Spot = 110.0000
Volatility = 0.2000
Rate = 0.0800
Dividend = 0.1200

Strike Expiry Option Price
100.0000 0.2500 10.3340