NAG Toolbox for Matlab

s30sa

1 Purpose
s30sa computes the Asian geometric continuous average-rate option price.

2 Syntax
\[ [p, ifail] = s30sa(calput, x, s, t, sigma, r, b, 'm', m, 'n', n) \]

3 Description
s30sa computes the price of an Asian geometric continuous average-rate option for constant volatility, \( \sigma \), risk-free rate, \( r \), and cost of carry, \( b \) (see Kemna and Vorst (1990)). For a given strike price, \( X \), the price of a call option with underlying price, \( S \), and time to expiry, \( T \), is
\[
P_{\text{call}} = Se^{(b-r)T} \Phi(d_1) - Xe^{-rT} \Phi(d_2),
\]
and the corresponding put option price is
\[
P_{\text{put}} = Xe^{-rT} \Phi(-d_2) - Se^{(b-r)T} \Phi(-d_1),
\]
where
\[
d_1 = \frac{\ln(S/X) + (b + \sigma^2/2)T}{\sigma\sqrt{T}}
\]
and
\[
d_2 = d_1 - \sigma\sqrt{T},
\]
with
\[
\sigma = \frac{\sigma}{\sqrt{3}}, \quad b = \frac{1}{2} \left( b - \frac{\sigma^2}{6} \right).
\]
\( \Phi \) is the cumulative Normal distribution function,
\[
\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-y^2/2\right)dy.
\]

4 References

5 Parameters
5.1 Compulsory Input Parameters
1:  calput – string
    Determines whether the option is a call or a put.
    calput = 'C'
    A call. The holder has a right to buy.
calput = 'P'
   A put. The holder has a right to sell.

Constraint: calput = 'C' or 'P'.

2:  
   x(m) – double array

   m, the dimension of the array, must satisfy the constraint m ≥ 1.
   
   x(i) must contain X_i, the i\textsuperscript{th} strike price, for i = 1, 2, \ldots, m.
   
   Constraint: x(i) ≥ z and x(i) ≤ 1/z, where z = X02AMF(), the safe range parameter, for i = 1, 2, \ldots, m.

3:  
   s – double scalar

   S, the price of the underlying asset.

   Constraint: s ≥ z and s ≤ 1/z, where z = X02AMF(), the safe range parameter.

4:  
   t(n) – double array

   n, the dimension of the array, must satisfy the constraint n ≥ 1.
   
   t(i) must contain T_i, the i\textsuperscript{th} time, in years, to expiry, for i = 1, 2, \ldots, n.
   
   Constraint: t(i) ≥ z, where z = X02AMF(), the safe range parameter, for i = 1, 2, \ldots, n.

5:  
   sigma – double scalar

   \sigma, the volatility of the underlying asset. Note that a rate of 15% should be entered as 0.15.

   Constraint: sigma > 0.0.

6:  
   r – double scalar

   r, the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.

   Constraint: r ≥ 0.0.

7:  
   b – double scalar

   b, the annual cost of carry rate. Note that a rate of 8% should be entered as 0.08.

5.2 Optional Input Parameters

1:  
   m – int32 scalar

   Default: The dimension of the array x.
   the number of strike prices to be used.

   Constraint: m ≥ 1.

2:  
   n – int32 scalar

   Default: The dimension of the array t.
   the number of times to expiry to be used.

   Constraint: n ≥ 1.

5.3 Input Parameters Omitted from the MATLAB Interface

ldp
5.4 Output Parameters

1: \( p(ldp,n) \) – double array
   The \( m \times n \) array \( p \) contains the computed option prices.

2: \( ifail \) – int32 scalar
   \( ifail = 0 \) unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

\( ifail = 1 \)
   On entry, \( calput \neq 'C' \) or \( 'P' \).

\( ifail = 2 \)
   On entry, \( m \leq 0 \).

\( ifail = 3 \)
   On entry, \( n \leq 0 \).

\( ifail = 4 \)
   On entry, \( x(i) < z \) or \( x(i) > 1/z \), where \( z = \text{X02AMF()} \), the safe range parameter.

\( ifail = 5 \)
   On entry, \( s < z \) or \( s > 1/z \), where \( z = \text{X02AMF()} \), the safe range parameter.

\( ifail = 6 \)
   On entry, \( t(i) < z \), where \( z = \text{X02AMF()} \), the safe range parameter.

\( ifail = 7 \)
   On entry, \( sigma \leq 0.0 \).

\( ifail = 8 \)
   On entry, \( r < 0.0 \).

\( ifail = 11 \)
   On entry, \( ldp < m \).

7 Accuracy

The accuracy of the output is dependent on the accuracy of the cumulative Normal distribution function, \( \Phi \). This is evaluated using a rational Chebyshev expansion, chosen so that the maximum relative error in the expansion is of the order of the machine precision (see s15ab and s15ad). An accuracy close to machine precision can generally be expected.

8 Further Comments

None.
9 Example

```
put = 'P';
s = 80.0;
sigma = 0.2;
r = 0.05;
b = 0.08;
x = [85.0];
t = [0.25];

[p, ifail] = s30sa(put, x, s, t, sigma, r, b);

fprintf('Asian Option: Geometric Continuous Average-Rate
Asian Put:
');
fprintf(' Spot = %9.4f
', s);
fprintf(' Volatility = %9.4f
', sigma);
fprintf(' Rate = %9.4f
', r);
fprintf(' Cost of carry = %9.4f
', b);

fprintf(' Strike Expiry Option Price
');
for i=1:1
    for j=1:1
        fprintf('%9.4f %9.4f %9.4f
', x(i), t(j), p(i,j));
    end
end

Asian Option: Geometric Continuous Average-Rate
Asian Put:
   Spot     = 80.0000
   Volatility = 0.2000
   Rate     = 0.0500
   Cost of carry = 0.0800

   Strike Expiry Option Price
85.0000  0.2500  4.6922
```

Asian Option: Geometric Continuous Average-Rate
Asian Put:
   Spot     = 80.0000
   Volatility = 0.2000
   Rate     = 0.0500
   Cost of carry = 0.0800

   Strike Expiry Option Price
85.0000  0.2500  4.6922