1 Purpose

`s30sb` computes the Asian geometric continuous average-rate option price together with its sensitivities (Greeks).

2 Syntax

```matlab
[p, delta, gamma, vega, theta, rho, crho, vanna, charm, speed, colour, zomma, vomma, ifail] = s30sb(calput, x, s, t, sigma, r, b, 'm', m, 'n', n)
```

3 Description

`s30sb` computes the price of an Asian geometric continuous average-rate option, together with the Greeks or sensitivities, which are the partial derivatives of the option price with respect to certain of the other input parameters. The annual volatility, $\sigma$, risk-free rate, $r$, and cost of carry, $b$, are constants (see Kemna and Vorst (1990)). For a given strike price, $X$, the price of a call option with underlying price, $S$, and time to expiry, $T$, is

\[
P_{\text{call}} = S e^{(b - r)T} \Phi(\tilde{d}_1) - X e^{-rt} \Phi(\tilde{d}_2),
\]

and the corresponding put option price is

\[
P_{\text{put}} = X e^{-rt} \Phi(-\tilde{d}_2) - S e^{(b - r)T} \Phi(-\tilde{d}_1),
\]

where

\[
\tilde{d}_1 = \frac{\ln(S/X) + (b + \tilde{\sigma}^2/2)T}{\tilde{\sigma}\sqrt{T}}
\]

and

\[
\tilde{d}_2 = \tilde{d}_1 - \tilde{\sigma}\sqrt{T},
\]

with

\[
\tilde{\sigma} = \frac{\sigma}{\sqrt{3}}, \quad \tilde{b} = \frac{1}{2} \left( b - \frac{\sigma^2}{6} \right).
\]

$\Phi$ is the cumulative Normal distribution function,

\[
\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-y^2/2\right) dy.
\]

4 References

5 Parameters

5.1 Compulsory Input Parameters

1: \texttt{calput} – string

Determines whether the option is a call or a put.

\texttt{calput} = 'C'

A call. The holder has a right to buy.

\texttt{calput} = 'P'

A put. The holder has a right to sell.

Constraint: \texttt{calput} = 'C' or 'P'.

2: \texttt{x(m)} – double array

\texttt{x(i)} must contain \( X_i \), the \( i \)th strike price, for \( i = 1, 2, \ldots, m \).

Constraint: \texttt{x(i)} \geq z and \texttt{x(i)} \leq 1/z, where \( z = X02AMF() \), the safe range parameter, for \( i = 1, 2, \ldots, m \).

3: \texttt{s} – double scalar

\( S \), the price of the underlying asset.

Constraint: \texttt{s} \geq z and \texttt{s} \leq 1/z, where \( z = X02AMF() \), the safe range parameter.

4: \texttt{t(n)} – double array

\texttt{t(i)} must contain \( T_i \), the \( i \)th time, in years, to expiry, for \( i = 1, 2, \ldots, n \).

Constraint: \texttt{t(i)} \geq z, where \( z = X02AMF() \), the safe range parameter, for \( i = 1, 2, \ldots, n \).

5: \texttt{sigma} – double scalar

\( \sigma \), the volatility of the underlying asset. Note that a rate of 15\% should be entered as 0.15.

Constraint: \texttt{sigma} > 0.0.

6: \texttt{r} – double scalar

\( r \), the annual risk-free interest rate, continuously compounded. Note that a rate of 5\% should be entered as 0.05.

Constraint: \texttt{r} \geq 0.0.

7: \texttt{b} – double scalar

\( b \), the annual cost of carry rate. Note that a rate of 8\% should be entered as 0.08.

5.2 Optional Input Parameters

1: \texttt{m} – int32 scalar

Default: The dimension of the array \texttt{x}.

the number of strike prices to be used.

Constraint: \texttt{m} \geq 1.

2: \texttt{n} – int32 scalar

Default: The dimension of the array \texttt{t}. 

\texttt{m} = \texttt{int32 scalar}

\texttt{m} = \texttt{int32 scalar}

\texttt{n} = \texttt{int32 scalar}
the number of times to expiry to be used.

Constraint: \( n \geq 1 \).

**5.3 Input Parameters Omitted from the MATLAB Interface**

**ldp**

**5.4 Output Parameters**

1: \( p(ldp, n) \) – double array
   The \( m \times n \) array \( p \) contains the computed option prices.

2: \( \delta(ldp, n) \) – double array
   The \( m \times n \) array \( \delta \) contains the sensitivity, \( \frac{\partial P}{\partial S} \), of the option price to change in the price of the underlying asset.

3: \( \gamma(ldp, n) \) – double array
   The \( m \times n \) array \( \gamma \) contains the sensitivity, \( \frac{\partial^2 P}{\partial S^2} \), of \( \delta \) to change in the price of the underlying asset.

4: \( \nu(ldp, n) \) – double array
   The \( m \times n \) array \( \nu \) contains the sensitivity, \( \frac{\partial P}{\partial \sigma} \), of the option price to change in the volatility of the underlying asset.

5: \( \theta(ldp, n) \) – double array
   The \( m \times n \) array \( \theta \) contains the sensitivity, \( -\frac{\partial P}{\partial T} \), of the option price to change in the time to expiry of the option.

6: \( \rho(ldp, n) \) – double array
   The \( m \times n \) array \( \rho \) contains the sensitivity, \( \frac{\partial P}{\partial r} \), of the option price to change in the annual risk-free interest rate.

7: \( \sigma(ldp, n) \) – double array
   The \( m \times n \) array \( \sigma \) containing the sensitivity, \( \frac{\partial P}{\partial b} \), of the option price to change in the annual cost of carry rate, \( b \).

8: \( \nu(ldp, n) \) – double array
   The \( m \times n \) array \( \nu \) contains the sensitivity, \( \frac{\partial^2 P}{\partial S \partial \sigma} \), of \( \nu \) to change in the price of the underlying asset or, equivalently, the sensitivity of \( \delta \) to change in the volatility of the asset price.

9: \( \chi(ldp, n) \) – double array
   The \( m \times n \) array \( \chi \) contains the sensitivity, \( -\frac{\partial^2 P}{\partial S \partial T} \), of \( \delta \) to change in the time to expiry of the option.

10: \( \nu(ldp, n) \) – double array
    The \( m \times n \) array \( \nu \) contains the sensitivity, \( \frac{\partial^3 P}{\partial S^3} \), of \( \gamma \) to change in the price of the underlying asset.
11: **colour(ldp,n) – double array**
The $m \times n$ array `colour` contains the sensitivity, $-\frac{\partial V}{\partial \Delta}$, of `gamma` to change in the time to expiry of the option.

12: **zomma(ldp,n) – double array**
The $m \times n$ array `zomma` contains the sensitivity, $\frac{\partial V}{\partial \sigma}$, of `gamma` to change in the volatility of the underlying asset.

13: **vomma(ldp,n) – double array**
The $m \times n$ array `vomma` contains the sensitivity, $\frac{\partial^2 V}{\partial \sigma^2}$, of `vega` to change in the volatility of the underlying asset.

14: **ifail – int32 scalar**
`ifail = 0` unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

```
ifail = 1
   On entry, calput 6¼ 'C' or 'P'.

ifail = 2
   On entry, m ≤ 0.

ifail = 3
   On entry, n ≤ 0.

ifail = 4
   On entry, x(i) < z or x(i) > 1/z, where z = X02AMF(), the safe range parameter.

ifail = 5
   On entry, s < z or s > 1/z, where z = X02AMF(), the safe range parameter.

ifail = 6
   On entry, t(i) < z, where z = X02AMF(), the safe range parameter.

ifail = 7
   On entry, sigma ≤ 0.0.

ifail = 8
   On entry, r < 0.0.

ifail = 11
   On entry, ldp < m.
```

7 Accuracy

The accuracy of the output is dependent on the accuracy of the cumulative Normal distribution function, $\Phi$. This is evaluated using a rational Chebyshev expansion, chosen so that the maximum relative error in the
expansion is of the order of the machine precision (see s15ab and s15ad). An accuracy close to machine precision can generally be expected.

8 Further Comments
None.

9 Example

```matlab
put = 'C';
s = 80.0;
sigma = 0.2;
r = 0.05;
b = 0.08;
x = [97.0];
t = [0.25];

[p, delta, gamma, vega, theta, rho, crho, vanna, charm, speed, colour, ...
  zomma, vomma, ifail] = s30sb(put, x, s, t, sigma, r, b);

fprintf('
Asian Option: Geometric Continuous Average-Rate
Asian Call:
');
fprintf(' Spot = %9.4f
', s);
fprintf(' Volatility = %9.4f
', sigma);
fprintf(' Rate = %9.4f
', r);
fprintf(' Cost of carry = %9.4f

', b);
fprintf(' Time to Expiry : %8.4f
', t(1));
fprintf(' Strike Price Delta Gamma Vega Theta Rho CRho
');
fprintf('%8.4f %8.4f %8.4f %8.4f %8.4f %8.4f %8.4f
', x(1), ...
  p(1,1), delta(1,1), gamma(1,1), vega(1,1), theta(1,1), rho(1,1),
  crho(1,1));

fprintf('
 Strike Price Vanna Charm Speed Colour Zomma
');
fprintf('%8.4f %8.4f %8.4f %8.4f %8.4f %8.4f %8.4f
', x(1), ...
  p(1,1), vanna(1,1), charm(1,1), speed(1,1), colour(1,1),
  zomma(1,1), vomma(1,1));
```

Asian Option: Geometric Continuous Average-Rate
Asian Call:

<table>
<thead>
<tr>
<th>Spot</th>
<th>80.0000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility</td>
<td>0.2000</td>
</tr>
<tr>
<td>Rate</td>
<td>0.0500</td>
</tr>
<tr>
<td>Cost of carry</td>
<td>0.0800</td>
</tr>
</tbody>
</table>

Time to Expiry : 0.2500

Strike Price Delta Gamma Vega Theta Rho CRho

<table>
<thead>
<tr>
<th>97.0000</th>
<th>0.0010</th>
<th>0.0008</th>
<th>0.0006</th>
<th>0.0638</th>
<th>-0.0281</th>
<th>0.0079</th>
<th>0.0081</th>
</tr>
</thead>
</table>
| Strike Price Vanna Charm Speed Colour Zomma Vomma

| 97.0000 | 0.0010 | 0.0443 | -0.0196 | 0.0004 | -0.0122 | 0.0272 | 3.3893 |