

# NAG Library Function Document

## nag\_ztfsm (f16zlc)

### 1 Purpose

nag\_ztfsm (f16zlc) performs one of the matrix-matrix operations

$$\begin{aligned} B &\leftarrow \alpha A^{-1}B, & B &\leftarrow \alpha A^{-H}B, \\ B &\leftarrow \alpha BA^{-1} \quad \text{or} & B &\leftarrow \alpha BA^{-H}, \end{aligned}$$

where  $A$  is a complex triangular matrix stored in Rectangular Full Packed (RFP) format,  $B$  is an  $m$  by  $n$  complex matrix, and  $\alpha$  is a complex scalar.  $A^{-H}$  denotes  $(A^H)^{-1}$  or equivalently  $(A^{-1})^H$ .

No test for singularity or near-singularity of  $A$  is included in this function. Such tests must be performed before calling this function.

### 2 Specification

```
#include <nag.h>
#include <nagf16.h>

void nag_ztfsm (Nag_OrderType order, Nag_RFP_Store transr,
               Nag_SideType side, Nag_UploType uplo, Nag_TransType trans,
               Nag_DiagType diag, Integer m, Integer n, Complex alpha,
               const Complex ar[], Complex b[], Integer pdb, NagError *fail)
```

### 3 Description

nag\_ztfsm (f16zlc) solves (for  $X$ ) a triangular linear system of one of the forms

$$\begin{aligned} AX &= \alpha B, & A^H X &= \alpha B, \\ XA &= \alpha B \quad \text{or} & XA^H &= \alpha B, \end{aligned}$$

where  $A$  is a complex triangular matrix stored in RFP format,  $B$ ,  $X$  are  $m$  by  $n$  complex matrices, and  $\alpha$  is a complex scalar. The RFP storage format is described in Section 3.3.3 in the f07 Chapter Introduction.

### 4 References

Gustavson F G, Waśniewski J, Dongarra J J and Langou J (2010) Rectangular full packed format for Cholesky's algorithm: factorization, solution, and inversion *ACM Trans. Math. Software* **37**, 2

### 5 Arguments

1: **order** – Nag\_OrderType *Input*

*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.

- 2:     **transr** – Nag\_RFP\_Store *Input*  
*On entry:* specifies whether the normal RFP representation of  $A$  or its conjugate transpose is stored.  
**transr** = Nag\_RFP\_Normal  
The matrix  $A$  is stored in normal RFP format.  
**transr** = Nag\_RFP\_ConjTrans  
The conjugate transpose of the RFP representation of the matrix  $A$  is stored.  
*Constraint:* **transr** = Nag\_RFP\_Normal or Nag\_RFP\_ConjTrans.
- 3:     **side** – Nag\_SideType *Input*  
*On entry:* specifies whether  $B$  is operated on from the left or the right, or similarly whether  $A$  (or its transpose) appears to the left or right of the solution matrix in the linear system to be solved.  
**side** = Nag\_LeftSide  
 $B$  is pre-multiplied from the left. The system to be solved has the form  $AX = \alpha B$  or  $A^T X = \alpha B$ .  
**side** = Nag\_RightSide  
 $B$  is post-multiplied from the right. The system to be solved has the form  $XA = \alpha B$  or  $XA^T = \alpha B$ .  
*Constraint:* **side** = Nag\_LeftSide or Nag\_RightSide.
- 4:     **uplo** – Nag\_UploType *Input*  
*On entry:* specifies whether  $A$  is upper or lower triangular.  
**uplo** = Nag\_Upper  
 $A$  is upper triangular.  
**uplo** = Nag\_Lower  
 $A$  is lower triangular.  
*Constraint:* **uplo** = Nag\_Upper or Nag\_Lower.
- 5:     **trans** – Nag\_TransType *Input*  
*On entry:* specifies whether the operation involves  $A^{-1}$  or  $A^{-H}$ , i.e., whether or not  $A$  is transpose conjugated in the linear system to be solved.  
**trans** = Nag\_NoTrans  
The operation involves  $A^{-1}$ , i.e.,  $A$  is not transpose conjugated.  
**trans** = Nag\_ConjTrans  
The operation involves  $A^{-H}$ , i.e.,  $A$  is transpose conjugated.  
*Constraint:* **trans** = Nag\_NoTrans or Nag\_ConjTrans.
- 6:     **diag** – Nag\_DiagType *Input*  
*On entry:* specifies whether  $A$  has nonunit or unit diagonal elements.  
**diag** = Nag\_NonUnitDiag  
The diagonal elements of  $A$  are stored explicitly.  
**diag** = Nag\_UnitDiag  
The diagonal elements of  $A$  are assumed to be 1, the corresponding elements of **ar** are not referenced.  
*Constraint:* **diag** = Nag\_NonUnitDiag or Nag\_UnitDiag.

- 7: **m** – Integer *Input*  
*On entry:*  $m$ , the number of rows of the matrix  $B$ .  
*Constraint:*  $\mathbf{m} \geq 0$ .
- 8: **n** – Integer *Input*  
*On entry:*  $n$ , the number of columns of the matrix  $B$ .  
*Constraint:*  $\mathbf{n} \geq 0$ .
- 9: **alpha** – Complex *Input*  
*On entry:* the scalar  $\alpha$ .
- 10: **ar**[*dim*] – const Complex *Input*  
**Note:** the dimension, *dim*, of the array **ar** must be at least  
 $\max(1, \mathbf{m} \times (\mathbf{m} + 1)/2)$  when **side** = Nag\_LeftSide;  
 $\max(1, \mathbf{n} \times (\mathbf{n} + 1)/2)$  when **side** = Nag\_RightSide.  
*On entry:* the  $m$  by  $m$  triangular matrix  $A$  if **side** = Nag\_LeftSide or the  $n$  by  $n$  triangular matrix  $A$  if **side** = Nag\_RightSide, stored in RFP format (as specified by **transr**). The storage format is described in detail in Section 3.3.3 in the f07 Chapter Introduction. If **alpha** = 0.0, **ar** is not referenced.
- 11: **b**[*dim*] – Complex *Input/Output*  
**Note:** the dimension, *dim*, of the array **b** must be at least  
 $\max(1, \mathbf{pdb} \times \mathbf{n})$  when **order** = Nag\_ColMajor;  
 $\max(1, \mathbf{m} \times \mathbf{pdb})$  when **order** = Nag\_RowMajor.  
*On entry:* the  $m$  by  $n$  matrix  $B$ .  
If **alpha** = 0, **b** need not be set.  
*On exit:* the updated matrix  $B$ , or similarly the solution matrix  $X$ .  
If **order** = Nag\_ColMajor,  $B_{ij}$  is stored in  $\mathbf{b}[(j - 1) \times \mathbf{pdb} + i - 1]$ .  
If **order** = Nag\_RowMajor,  $B_{ij}$  is stored in  $\mathbf{b}[(i - 1) \times \mathbf{pdb} + j - 1]$ .
- 12: **pdb** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **b**.  
*Constraints:*  
if **order** = Nag\_ColMajor,  $\mathbf{pdb} \geq \max(1, \mathbf{m})$ ;  
if **order** = Nag\_RowMajor,  $\mathbf{pdb} \geq \max(1, \mathbf{n})$ .
- 13: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

**NE\_BAD\_PARAM**

On entry, argument  $\langle value \rangle$  had an illegal value.

**NE\_INT**

On entry,  $\mathbf{m} = \langle value \rangle$ .

Constraint:  $\mathbf{m} \geq 0$ .

On entry,  $\mathbf{n} = \langle value \rangle$ .

Constraint:  $\mathbf{n} \geq 0$ .

**NE\_INT\_2**

On entry,  $\mathbf{pdb} = \langle value \rangle$ ,  $\mathbf{m} = \langle value \rangle$ .

Constraint:  $\mathbf{pdb} \geq \max(1, \mathbf{m})$ .

On entry,  $\mathbf{pdb} = \langle value \rangle$  and  $\mathbf{n} = \langle value \rangle$ .

Constraint:  $\mathbf{pdb} \geq \max(1, \mathbf{n})$ .

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

**7 Accuracy**

Not applicable.

**8 Parallelism and Performance**

nag\_ztfsn (f16zlc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

**9 Further Comments**

None.

**10 Example**

This example reads in the upper triangular part of a symmetric matrix  $A$  which it converts to RFP format. It also reads in  $\alpha$  and a 4 by 3 matrix  $B$  and then performs the matrix-matrix operation  $B \leftarrow \alpha A^{-1} B$ .

## 10.1 Program Text

```

/* nag_ztfsm (f16zlc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <nag_stdlib.h>
#include <nagf01.h>
#include <nagf16.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    Integer exit_status = 0;
    Complex alpha;
    Integer i, j, m, n, pda, pdb;
    /* Arrays */
    Complex *a = 0, *ar = 0, *b = 0;
    char nag_enum_arg[40];
    /* Nag Types */
    Nag_OrderType order;
    Nag_RFP_Store transr;
    Nag_SideType side;
    Nag_UploType uplo;
    Nag_TransType trans;
    NagError fail;

    INIT_FAIL(fail);

    printf("nag_ztfsm (f16zlc) Example Program Results\n");
    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &m, &n);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &m, &n);
#endif
    pda = m;

#ifdef NAG_COLUMN_MAJOR
    order = Nag_ColMajor;
    pdb = m;
#define A(I, J) a[(J-1)*pda + I-1]
#define B(I, J) b[(J-1)*pdb + I-1]
#else
    order = Nag_RowMajor;
    pdb = n;
#define A(I, J) a[(I-1)*pda + J-1]
#define B(I, J) b[(I-1)*pdb + J-1]
#endif

    if (!(a = NAG_ALLOC(pda * m, Complex)) ||
        !(ar = NAG_ALLOC((m * (m + 1)) / 2, Complex)) ||
        !(b = NAG_ALLOC(m * n, Complex)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
}

```

```

    /* Nag_RFP_Store */
#ifdef _WIN32
    scanf_s("%39s ", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf("%39s ", nag_enum_arg);
#endif
    transr = (Nag_RFP_Store) nag_enum_name_to_value(nag_enum_arg);
    /* Nag_SideType */
#ifdef _WIN32
    scanf_s("%39s  %*[\n] ", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf("%39s  %*[\n] ", nag_enum_arg);
#endif
    side = (Nag_SideType) nag_enum_name_to_value(nag_enum_arg);
    /* Nag_UploType */
#ifdef _WIN32
    scanf_s("%39s ", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf("%39s ", nag_enum_arg);
#endif
    uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);
    /* Nag_TransType */
#ifdef _WIN32
    scanf_s("%39s  %*[\n] ", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf("%39s  %*[\n] ", nag_enum_arg);
#endif
    trans = (Nag_TransType) nag_enum_name_to_value(nag_enum_arg);
#ifdef _WIN32
    scanf_s(" ( %lf , %lf ) %*[\n] ", &alpha.re, &alpha.im);
#else
    scanf(" ( %lf , %lf ) %*[\n] ", &alpha.re, &alpha.im);
#endif
    /* Read upper or lower triangle of matrix A from data file */
    if (uplo == Nag_Lower) {
        for (i = 1; i <= m; i++) {
            for (j = 1; j <= i; j++) {
#ifdef _WIN32
                scanf_s(" ( %lf , %lf ) ", &A(i, j).re, &A(i, j).im);
#else
                scanf(" ( %lf , %lf ) ", &A(i, j).re, &A(i, j).im);
#endif
            }
        }
    }
    else {
        for (i = 1; i <= m; i++) {
            for (j = i; j <= m; j++) {
#ifdef _WIN32
                scanf_s(" ( %lf , %lf ) ", &A(i, j).re, &A(i, j).im);
#else
                scanf(" ( %lf , %lf ) ", &A(i, j).re, &A(i, j).im);
#endif
            }
        }
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    /* Read matrix B from data file */
    for (i = 1; i <= m; i++) {
        for (j = 1; j <= n; j++) {
#ifdef _WIN32
            scanf_s(" ( %lf , %lf ) ", &B(i, j).re, &B(i, j).im);
#else
            scanf(" ( %lf , %lf ) ", &B(i, j).re, &B(i, j).im);
#endif
        }
    }

```

```

}
/* Convert complex triangular matrix A from full to rectangular full packed
 * storage format (stored in ar) using nag_ztrttf (f01vfc).
 */
nag_ztrttf(order, transr, uplo, m, a, pda, ar, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_ztrttf (f01vfc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

printf("\n");
/* Solve AX = B, where complex triangular matrix A is stored using RFP format
 * in ar, using nag_ztfsm (f16zlc).
 */
nag_ztfsm(order, transr, side, uplo, trans, Nag_NonUnitDiag, m, n, alpha,
          ar, b, pdb, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_ztfsm (f16zlc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Print the result using easy-to-use complex general matrix printing routine
 * nag_gen_complx_mat_print (x04dac).
 */
fflush(stdout);
nag_gen_complx_mat_print(order, Nag_GeneralMatrix, Nag_NonUnitDiag, m, n, b,
                        pdb, "The Solution", 0, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_complx_mat_print (x04dac).\n%s\n",
          fail.message);
    exit_status = 1;
}

END:
    NAG_FREE(a);
    NAG_FREE(ar);
    NAG_FREE(b);
    return exit_status;
}

```

## 10.2 Program Data

```

nag_ztfsm (f16zlc) Example Program Data
4 3 : m, n
Nag_RFP_Normal Nag_LeftSide : transr, side
Nag_Upper Nag_NoTrans : uplo, trans
(4.21,1.28) : alpha
(1.1,1.1) (1.2,1.2) (1.3,1.3) (1.4,1.4)
(2.2,2.2) (2.3,2.3) (2.4,2.4)
(3.3,3.3) (3.4,3.4)
(4.4,4.4) : matrix A
( 1.80,0.59) ( 2.88, 1.23) (2.05, 0.78)
( 5.25,0.12) ( 1.76,-2.95) (2.20,-0.95)
( 1.58,2.01) (-2.69, 3.18) (0.11,-2.90)
(-1.11,1.11) (-0.66, 1.66) (1.59,-0.59) : matrix B

```

## 10.3 Program Results

nag\_ztfsm (f16zlc) Example Program Results

```

The Solution
      1      2      3
1  -2.0339  8.6009  3.8676
   2.6429  4.3188  2.2452

2   4.3280  1.0930  3.3517
  -4.3756 -8.8840 -0.0650

```

3	2.5393	-0.9711	-2.0155
	-0.1237	2.5460	-1.5364
4	-0.3229	0.1410	0.7955
	1.0621	1.2554	-0.8975

---