

NAG Library Routine Document

S01BAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

S01BAF returns a value of the shifted logarithmic function, $\ln(1+x)$, via the routine name.

2 Specification

```
double precision FUNCTION S01BAF(X, IFAIL)
INTEGER                                IFAIL
double precision                                X
```

3 Description

S01BAF computes values of $\ln(1+x)$, retaining full relative precision even when $|x|$ is small. The routine is based on the Chebyshev expansion

$$\ln \frac{1+p^2+2p\bar{x}}{1+p^2-2p\bar{x}} = 4 \sum_{k=0}^{\infty} \frac{p^{2k+1}}{2k+1} T_{2k+1}(\bar{x}).$$

Setting $\bar{x} = \frac{x(1+p^2)}{2p(x+2)}$, and choosing $p = \frac{q-1}{q+1}$, $q = \sqrt[4]{2}$ the expansion is valid in the domain $x \in \left[\frac{1}{\sqrt{2}} - 1, \sqrt{2} - 1 \right]$.

Outside this domain, $\ln(1+x)$ is computed by the standard logarithmic function.

4 References

Lyusternik L A, Chervonenkis O A and Yanpolskii A R (1965) *Handbook for Computing Elementary Functions* p. 57 Pergamon Press

5 Parameters

1: X – *double precision* *Input*

On entry: the argument x of the function.

Constraint: $X > -1.0$.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry $IFAIL = 0$ or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

$IFAIL = 1$

On entry, $X \leq -1.0$.

The result is returned as zero.

7 Accuracy

The returned result should be accurate almost to *machine precision*, with a limit of about 20 significant figures due to the precision of internal constants. Note however that if x lies very close to -1.0 and is not exact (for example if x is the result of some previous computation and has been rounded), then precision will be lost in the computation of $1 + x$, and hence $\ln(1 + x)$, in S01BAF.

8 Further Comments

Empirical tests show that the time taken for a call of S01BAF usually lies between about 1.25 and 2.5 times the time for a call to the standard logarithm function.

9 Example

The example program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*      S01BAF Example Program Text
*      Mark 14 Release. NAG Copyright 1989
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. External Functions ..
      DOUBLE PRECISION S01BAF
      EXTERNAL         S01BAF
*      .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S01BAF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '          X          Y'
20    READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S01BAF(X,IFAIL)
*
      IF (IFAIL.EQ.0) THEN
          WRITE (NOUT,99999) X, Y
          GO TO 20
      ELSE
          WRITE (NOUT,99998) IFAIL
      END IF
      40 CONTINUE
*
99999 FORMAT (1X,1P,2E12.4)
99998 FORMAT (1X,/1X,' ** S01BAF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

S01BAF Example Program Data
2.50E+0
1.25E-1
-9.06E-1
1.29E-3
-7.83E-6
1.00E-9

9.3 Program Results

S01BAF Example Program Results

X	Y
2.5000E+00	1.2528E+00
1.2500E-01	1.1778E-01
-9.0600E-01	-2.3645E+00
1.2900E-03	1.2892E-03
-7.8300E-06	-7.8300E-06
1.0000E-09	1.0000E-09
