NAG Library Routine Document

S07AAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S07AAF returns the value of the circular tangent, $\tan x$, via the routine name.

2 Specification

double precision `FUNCTION S07AAF(X, IFAIL)`

`INTEGER` `IFAIL`
double precision `X`

3 Description

S07AAF calculates an approximate value for the circular tangent of its argument, $\tan x$. It is based on the Chebyshev expansion

$$\tan\theta = \theta y(t) = \theta {\sum_{r=0}}' c_r T_r(t)$$

where $-\dfrac{\pi}{4} < \theta < \dfrac{\pi}{4}$ and $-1 < t < +1, \qquad t = 2\left(\dfrac{4\theta}{\pi}\right)^2 - 1.$

The reduction to the standard range is accomplished by taking

$$x = N\pi/2 + \theta$$

where N is an integer and $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$,

i.e., $\theta = x - \left(\dfrac{2x}{\pi}\right)\frac{\pi}{2}$ where $N = \left[\dfrac{2x}{\pi}\right] =$ the nearest integer to $\dfrac{2x}{\pi}$.

From the properties of $\tan x$ it follows that

$$\tan x = \left\{ \begin{matrix} \tan\theta, & N\text{even} \\ -1/\tan\theta, & N\text{odd} \end{matrix} \right\}$$

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – ***double precision*** *Input*

On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

The routine has been called with an argument that is larger in magnitude than F; the default result returned is zero. The value of F is given in the Users' Note for your implementation.

IFAIL $= 2$

The routine has been called with an argument that is too close (as determined using the relative tolerance F) to an odd multiple of $\pi/2$, at which the function is infinite; the routine returns a value with the correct sign but a more or less arbitrary but large magnitude (see Section 7). The value of F is given in the Users' Note for your implementation.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result respectively, then in principle

$$\epsilon \geq \frac{2x}{\sin 2x}\delta.$$

That is a relative error in the argument, x, is amplified by at least a factor $2x/\sin 2x$ in the result.

Similarly if E is the absolute error in the result this is given by

$$E \geq \frac{x}{\cos^2 x}\delta.$$

The equalities should hold if δ is greater than the ***machine precision*** (δ is a result of data errors etc.) but if δ is simply the round-off error in the machine it is possible that internal calculation rounding will lose an extra figure.

The graphs below show the behaviour of these amplification factors.

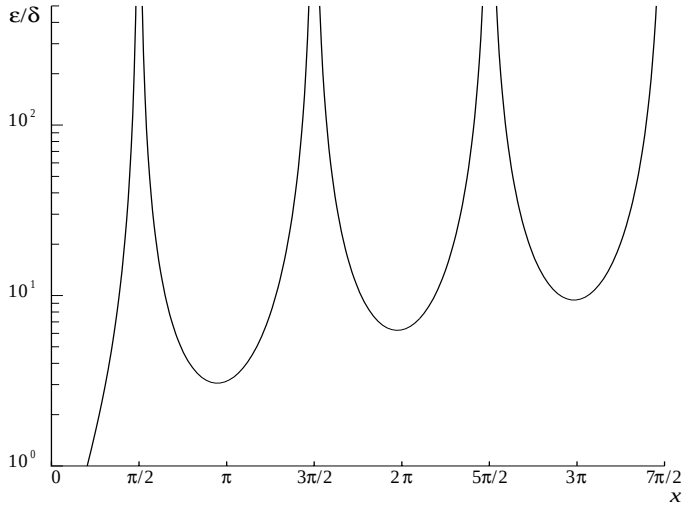

Figure 1

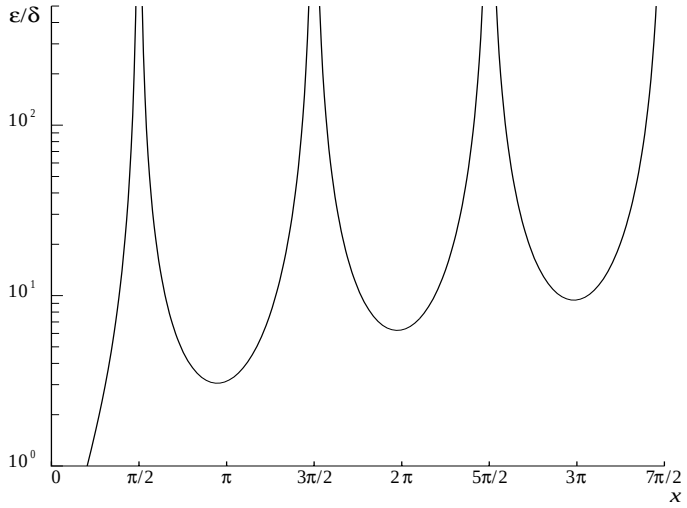

Figure 2

In the principal range it is possible to preserve relative accuracy even near the zero of $\tan x$ at $x = 0$ but at the other zeros only absolute accuracy is possible. Near the infinities of $\tan x$ both the relative and absolute errors become infinite and the routine must fail (error 2).

If N is odd and $|\theta| \leq xF_2$ the routine could not return better than two figures and in all probability would produce a result that was in error in its most significant figure. Therefore the routine fails and it returns the value

$$-\operatorname{sign}\theta\left(\frac{1}{|xF_2|}\right) \simeq -\operatorname{sign}\theta\tan\left(\frac{\pi}{2}-|xF_2|\right)$$

which is the value of the tangent at the nearest argument for which a valid call could be made.

Accuracy is also unavoidably lost if the routine is called with a large argument. If $|x| > F_1$ the routine fails (error 1) and returns zero. (See the Users' Note for your implementation for specific values of F_1 and F_2.)

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*     S07AAF Example Program Text
*     Mark 14 Revised. NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*     .. External Functions ..
      DOUBLE PRECISION S07AAF
      EXTERNAL         S07AAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S07AAF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
```

```
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X           Y         IFAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S07AAF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
         WRITE (NOUT,99999) X, Y, IFAIL
         GO TO 20
      ELSE
         WRITE (NOUT,99998) IFAIL
      END IF
   40 CONTINUE
*
99999 FORMAT (1X,1P,2E12.3,I7)
99998 FORMAT (1X,' ** S07AAF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

```
S07AAF Example Program Data
            -2.0
            -0.5
             1.0
             3.0
             1.5708
```

9.3 Program Results

```
 S07AAF Example Program Results

      X           Y         IFAIL

   -2.000E+00   2.185E+00      0
   -5.000E-01  -5.463E-01      0
    1.000E+00   1.557E+00      0
    3.000E+00  -1.425E-01      0
    1.571E+00  -2.722E+05      0
```