

NAG Library Routine Document

S09AAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

S09AAF returns the value of the inverse circular sine, $\arcsin x$, via the routine name. The value is in the principal range $(-\pi/2, \pi/2)$.

2 Specification

```

double precision FUNCTION S09AAF(X, IFAIL)
INTEGER                                IFAIL
double precision                                X

```

3 Description

S09AAF calculates an approximate value for the inverse circular sine, $\arcsin x$. It is based on the Chebyshev expansion

$$\arcsin x = x \times y(x) = x \sum_{r=0}^{\infty} a_r T_r(t)$$

where $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$ and $t = 4x^2 - 1$.

For $x^2 \leq \frac{1}{2}$, $\arcsin x = x \times y(x)$.

For $\frac{1}{2} < x^2 \leq 1$, $\arcsin x = \text{sign } x \left\{ \frac{\pi}{2} - \arcsin \sqrt{1 - x^2} \right\}$.

For $x^2 > 1$, $\arcsin x$ is undefined and the routine fails.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – **double precision** *Input*

On entry: the argument x of the function.

Constraint: $|X| \leq 1.0$.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the

recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

The routine has been called with an argument greater than 1.0 in absolute value; $\arcsin x$ is undefined and the routine returns zero.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\arcsin x \sqrt{1-x^2}} \times \delta \right|.$$

That is, a relative error in the argument x is amplified by at least a factor $\frac{x}{\arcsin x \sqrt{1-x^2}}$ in the result.

The equality should hold if δ is greater than the *machine precision* (δ is a result of data errors etc.) but if δ is produced simply by round-off error in the machine it is possible that rounding in internal calculations may lose an extra figure in the result.

This factor stays close to one except near $|x| = 1$ where its behaviour is shown in the following graph.

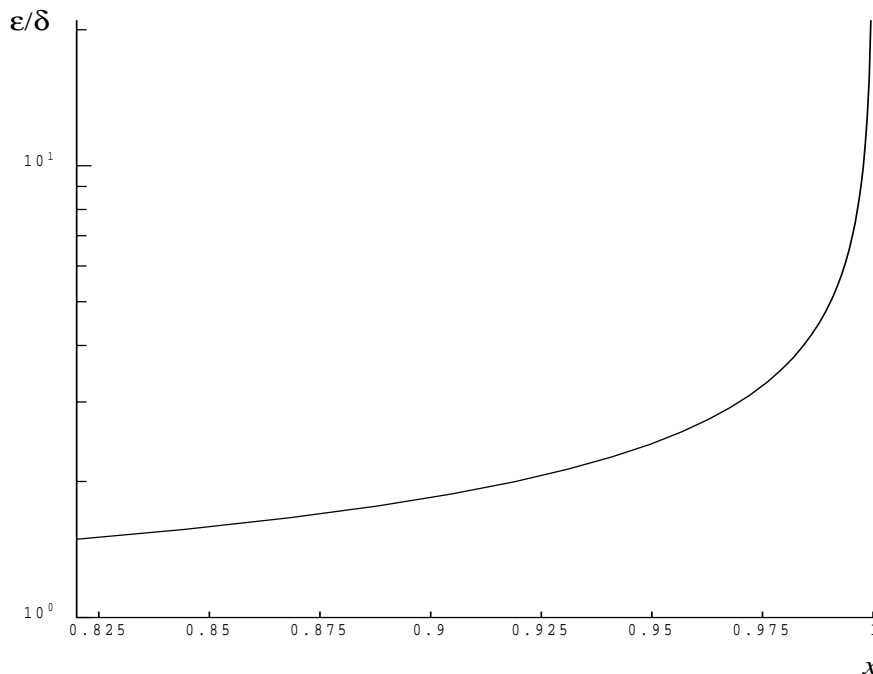


Figure 1

For $|x|$ close to unity, $1 - |x| \sim \delta$, the above analysis is no longer applicable owing to the fact that both argument and result are subject to finite bounds, ($|x| \leq 1$ and $|\arcsin x| \leq \frac{1}{2}\pi$). In this region $\epsilon \sim \sqrt{\delta}$; that is the result will have approximately half as many correct significant figures as the argument.

For $|x| = 1$ the result will be correct to full *machine precision*.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*      S09AAF Example Program Text
*      Mark 14 Revised. NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER       (NIN=5,NOUT=6)
*      .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*      .. External Functions ..
      DOUBLE PRECISION S09AAF
      EXTERNAL        S09AAF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S09AAF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '      X          Y          IFAIL'
      WRITE (NOUT,*)
20     READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S09AAF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
          WRITE (NOUT,99999) X, Y, IFAIL
          GO TO 20
      ELSE
          WRITE (NOUT,99998) IFAIL
      END IF
      40 CONTINUE
*
99999 FORMAT (1X,1P,2E12.3,I7)
99998 FORMAT (1X,' ** S09AAF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

```
S09AAF Example Program Data
      -0.5
      0.1
      0.9
      2.0
      -1.5
```

9.3 Program Results

```
S09AAF Example Program Results
```

X	Y	IFAIL
-5.000E-01	-5.236E-01	0
1.000E-01	1.002E-01	0
9.000E-01	1.120E+00	0
2.000E+00	0.000E+00	1
-1.500E+00	0.000E+00	1