

NAG Library Routine Document

S09ABF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

S09ABF returns the value of the inverse circular cosine, $\arccos x$, via the routine name; the result is in the principal range $(0, \pi)$.

2 Specification

double precision FUNCTION S09ABF(X, IFAIL)

INTEGER IFAIL

double precision X

3 Description

S09ABF calculates an approximate value for the inverse circular cosine, $\arccos x$. It is based on the Chebyshev expansion

$$\arcsin x = x \times y(t) = x \sum_{r=0}^{\infty} a_r T_r(t)$$

where $\frac{-1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$ and $t = 4x^2 - 1$.

For $x^2 \leq \frac{1}{2}$, $\arccos x = \frac{\pi}{2} - \arcsin x$.

For $-1 \leq x < \frac{-1}{\sqrt{2}}$, $\arccos x = \pi - \arcsin \sqrt{1 - x^2}$.

For $\frac{1}{\sqrt{2}} < x \leq 1$, $\arccos x = \arcsin \sqrt{1 - x^2}$.

For $|x| > 1$, $\arccos x$ is undefined and the routine fails.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – *double precision* *Input*

On entry: the argument x of the function.

Constraint: $|X| \leq 1.0$.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0 . **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry $IFAIL = 0$ or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

$IFAIL = 1$

S09ABF has been called with $|X| > 1.0$, for which \arccos is undefined. A zero result is returned.

7 Accuracy

If δ and ϵ are the relative errors in the argument and the result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\arccos x \sqrt{1-x^2}} \times \delta \right|.$$

The equality should hold if δ is greater than the *machine precision* (δ is due to data errors etc.), but if δ is due simply to round-off in the machine it is possible that rounding etc. in internal calculations may lose one extra figure.

The behaviour of the amplification factor $\frac{x}{\arccos x \sqrt{1-x^2}}$ is shown in the graph below.

In the region of $x = 0$ this factor tends to zero and the accuracy will be limited by the *machine precision*. For $|x|$ close to one, $1 - |x| \sim \delta$, the above analysis is not applicable owing to the fact that both the argument and the result are bounded $|x| \leq 1$, $0 \leq \arccos x \leq \pi$.

In the region of $x \sim -1$ we have $\epsilon \sim \sqrt{\delta}$, that is the result will have approximately half as many correct significant figures as the argument.

In the region $x \sim +1$, we have that the absolute error in the result, E , is given by $E \sim \sqrt{\delta}$, that is the result will have approximately half as many decimal places correct as there are correct figures in the argument.

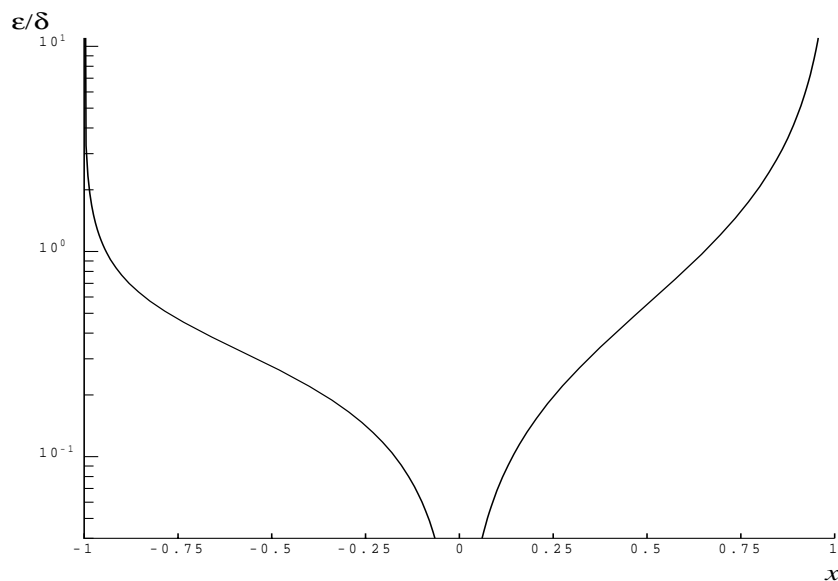


Figure 1

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*      S09ABF Example Program Text
*      Mark 14 Revised. NAG Copyright 1989.
*      .. Parameters ..
INTEGER          NIN, NOUT
PARAMETER       (NIN=5,NOUT=6)
*      .. Local Scalars ..
DOUBLE PRECISION X, Y
INTEGER         IFAIL
*      .. External Functions ..
DOUBLE PRECISION S09ABF
EXTERNAL        S09ABF
*      .. Executable Statements ..
WRITE (NOUT,*) 'S09ABF Example Program Results'
*      Skip heading in data file
READ (NIN,*)
WRITE (NOUT,*)
WRITE (NOUT,*) '      X          Y          IFAIL'
WRITE (NOUT,*)
20 READ (NIN,*,END=40) X
   IFAIL = 1
*
*      Y = S09ABF(X,IFAIL)
*
   IF (IFAIL.GE.0) THEN
       WRITE (NOUT,99999) X, Y, IFAIL
       GO TO 20
   ELSE
       WRITE (NOUT,99998) IFAIL
   END IF
```

```
40 CONTINUE
*
99999 FORMAT (1X,1P,2E12.3,I7)
99998 FORMAT (1X,' ** S09ABF returned with IFAIL = ',I5)
END
```

9.2 Program Data

```
S09ABF Example Program Data
      -0.5
       0.1
       0.9
       2.0
      -1.5
```

9.3 Program Results

```
S09ABF Example Program Results
```

X	Y	IFAIL
-5.000E-01	2.094E+00	0
1.000E-01	1.471E+00	0
9.000E-01	4.510E-01	0
2.000E+00	0.000E+00	1
-1.500E+00	0.000E+00	1
