NAG Library Routine Document

S10AAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S10AAF returns a value for the hyperbolic tangent, $\tanh x$, via the routine name.

2 Specification

double precision `FUNCTION S10AAF(X, IFAIL)`

`INTEGER` `IFAIL`
double precision `X`

3 Description

S10AAF calculates an approximate value for the hyperbolic tangent of its argument, $\tanh x$.

For $|x| \leq 1$ it is based on the Chebyshev expansion

$$\tanh x = x \times y(t) = x{\sum_{r=0}}' a_r T_r(t)$$

where $-1 \leq x \leq 1$, $-1 \leq t \leq 1$, and $t = 2x^2 - 1$.

For $1 < |x| < E_1$ (see the Users' Note for your implementation for value of E_1)

$$\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}.$$

For $|x| \geq E_1$, $\tanh x = \text{sign}\, x$ to within the representation accuracy of the machine and so this approximation is used.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – ***double precision*** *Input*

On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

None.

7 Accuracy

If δ and ϵ are the relative errors in the argument and the result respectively, then in principle,

$$|\epsilon| \simeq \left|\frac{2x}{\sinh 2x}\delta\right|.$$

That is, a relative error in the argument, x, is amplified by a factor approximately $\frac{2x}{\sinh 2x}$, in the result. The equality should hold if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is due simply to the round-off in the machine representation it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the amplification factor is shown in the following graph:

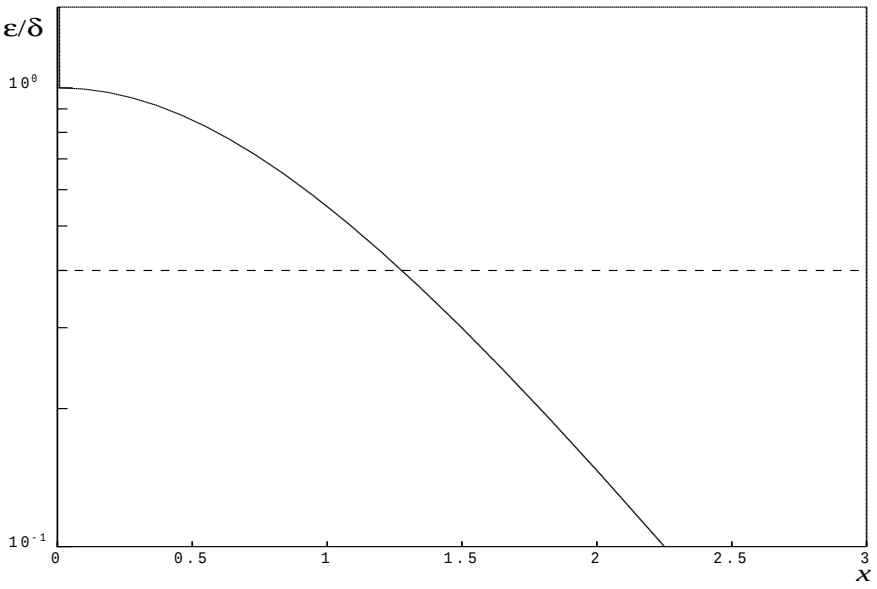

Figure 1

It should be noted that this factor is always less than or equal to 1.0 and away from $x = 0$ the accuracy will eventually be limited entirely by the precision of machine representation.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*     S10AAF Example Program Text
*     Mark 14 Revised. NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*     .. External Functions ..
      DOUBLE PRECISION S10AAF
      EXTERNAL         S10AAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S10AAF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X           Y        IFAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S10AAF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
         WRITE (NOUT,99999) X, Y, IFAIL
         GO TO 20
      ELSE
         WRITE (NOUT,99998) IFAIL
      END IF
   40 CONTINUE
*
99999 FORMAT (1X,1P,2E12.3,I7)
99998 FORMAT (1X,' ** S10AAF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

```
S10AAF Example Program Data
          -20.0
           -5.0
            0.5
            5.0
```

9.3 Program Results

```
 S10AAF Example Program Results

      X           Y        IFAIL

   -2.000E+01  -1.000E+00      0
   -5.000E+00  -9.999E-01      0
    5.000E-01   4.621E-01      0
    5.000E+00   9.999E-01      0
```