

NAG Library Routine Document

S10ABF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

S10ABF returns the value of the hyperbolic sine, $\sinh x$, via the routine name.

2 Specification

```

double precision FUNCTION S10ABF(X, IFAIL)
INTEGER                                IFAIL
double precision                                X

```

3 Description

S10ABF calculates an approximate value for the hyperbolic sine of its argument, $\sinh x$.

For $|x| \leq 1$ it uses the Chebyshev expansion

$$\sinh x = x \times y(t) = x \sum_{r=0}^{\prime} a_r T_r(t)$$

where $t = 2x^2 - 1$.

For $1 < |x| \leq E_1$, $\sinh x = \frac{1}{2}(e^x - e^{-x})$

where E_1 is a machine-dependent constant, details of which are given in the Users' Note for your implementation.

For $|x| > E_1$, the routine fails owing to the danger of setting overflow in calculating e^x . The result returned for such calls is $\sinh(\text{sign } x E_1)$, i.e., it returns the result for the nearest valid argument.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – *double precision* *Input*

On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry $IFAIL = 0$ or -1 , explanatory error messages are output on the current error message unit (as defined by $X04AAF$).

Errors or warnings detected by the routine:

$IFAIL = 1$

The routine has been called with an argument too large in absolute magnitude. There is a danger of setting overflow. The result is the value of \sinh at the closest argument for which a valid call could be made. (See Section 3 and the Users' Note for your implementation.)

7 Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq |x \coth x \times \delta|.$$

That is the relative error in the argument, x , is amplified by a factor, approximately $x \coth x$. The equality should hold if δ is greater than the *machine precision* (δ is a result of data errors etc.) but, if δ is simply a result of round-off in the machine representation of x , then it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the error amplification factor can be seen in the following graph:

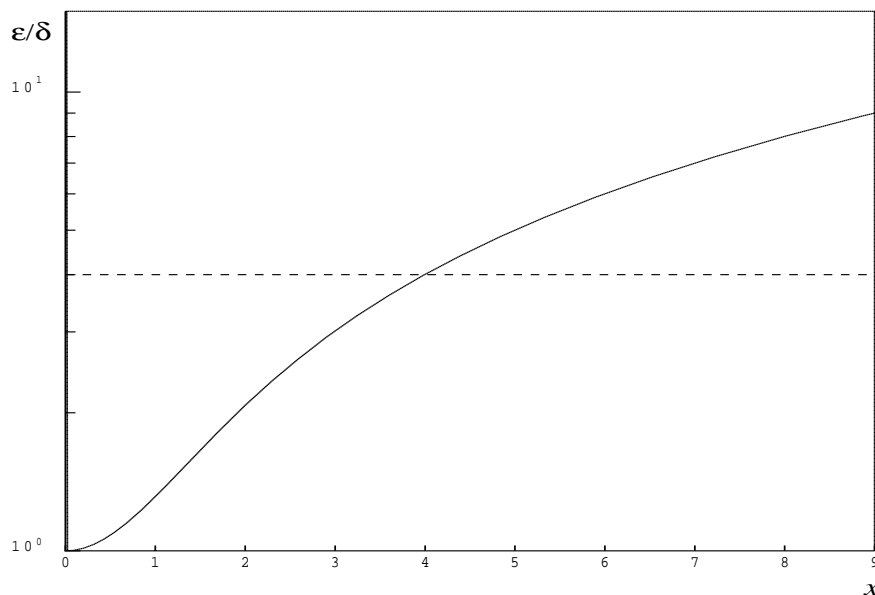


Figure 1

It should be noted that for $|x| \geq 2$

$$\epsilon \sim x\delta = \Delta$$

where Δ is the absolute error in the argument.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```

*      S10ABF Example Program Text
*      Mark 14 Revised. NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER       (NIN=5,NOUT=6)
*      .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*      .. External Functions ..
      DOUBLE PRECISION S10ABF
      EXTERNAL        S10ABF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S10ABF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '          X          Y          IFAIL'
      WRITE (NOUT,*)
20     READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S10ABF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
          WRITE (NOUT,99999) X, Y, IFAIL
          GO TO 20
      ELSE
          WRITE (NOUT,99998) IFAIL
      END IF
40     CONTINUE
*
99999  FORMAT (1X,1P,2E12.3,I7)
99998  FORMAT (1X,' ** S10ABF returned with IFAIL = ',I5)
      END

```

9.2 Program Data

```

S10ABF Example Program Data
      -10.0
      -0.5
      0.0
      0.5
      25.0

```

9.3 Program Results

S10ABF Example Program Results

X	Y	IFAIL
-1.000E+01	-1.101E+04	0
-5.000E-01	-5.211E-01	0
0.000E+00	0.000E+00	0
5.000E-01	5.211E-01	0
2.500E+01	3.600E+10	0
