NAG Library Routine Document

S11AAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S11AAF returns the value of the inverse hyperbolic tangent, $\operatorname{arctanh} x$, via the routine name.

2 Specification

double precision `FUNCTION S11AAF(X, IFAIL)`

```
INTEGER                  IFAIL
double precision         X
```

3 Description

S11AAF calculates an approximate value for the inverse hyperbolic tangent of its argument, $\operatorname{arctanh} x$.

For $x^2 \leq \frac{1}{2}$ it is based on the Chebyshev expansion

$$\operatorname{arctanh} x = x \times y(t) = x{\sum_{r=0}}' a_r T_r(t)$$

where $-\dfrac{1}{\sqrt{2}} \leq x \leq \dfrac{1}{\sqrt{2}}$,, $\quad -1 \leq t \leq 1$, $\quad$ and $\quad t = 4x^2 - 1$.

For $\frac{1}{2} < x^2 < 1$, it uses

$$\operatorname{arctanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right).$$

For $|x| \geq 1$, the routine fails as $\operatorname{arctanh} x$ is undefined.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – ***double precision*** *Input*

On entry: the argument x of the function.

Constraint: $|\mathrm{X}| < 1.0$.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the

recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

The routine has been called with an argument greater than or equal to 1.0 in magnitude, for which arctanh is not defined. The result is returned as zero.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{(1-x^2)\operatorname{arctanh} x} \times \delta \right|.$$

That is, the relative error in the argument, x, is amplified by at least a factor $\dfrac{x}{(1-x^2)\operatorname{arctanh} x}$ in the result. The equality should hold if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is simply due to round-off in the machine representation then it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the amplification factor is shown in the following graph:

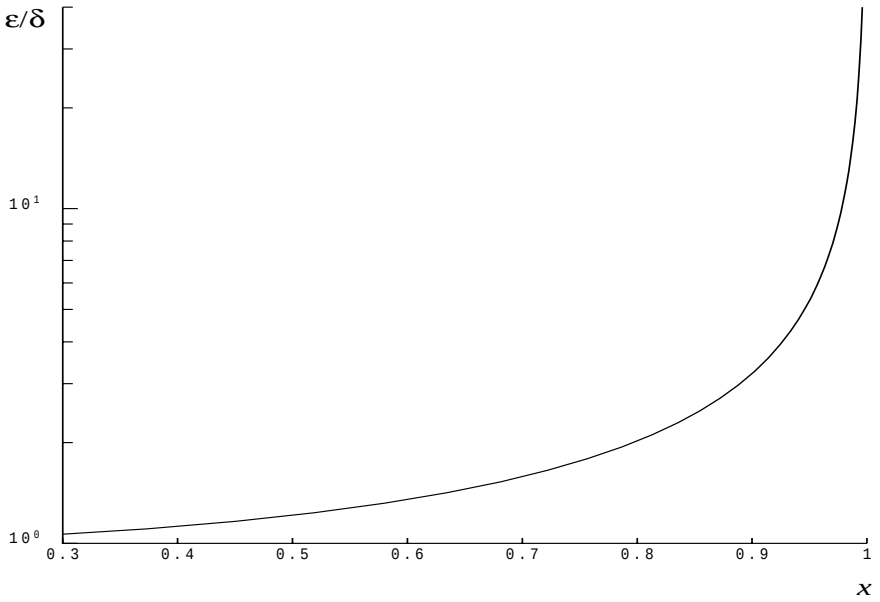

Figure 1

The factor is not significantly greater than one except for arguments close to $|x| = 1$. However in the region where $|x|$ is close to one, $1 - |x| \sim \delta$, the above analysis is inapplicable since x is bounded by definition, $|x| < 1$. In this region where arctanh is tending to infinity we have

$$\epsilon \sim 1/\ln \delta$$

which implies an obvious, unavoidable serious loss of accuracy near $|x| \sim 1$, e.g., if x and 1 agree to 6 significant figures, the result for $\operatorname{arctanh} x$ would be correct to at most about one figure.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*     S11AAF Example Program Text
*     Mark 14 Revised. NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*     .. External Functions ..
      DOUBLE PRECISION S11AAF
      EXTERNAL         S11AAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S11AAF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X           Y        IFAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S11AAF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
         WRITE (NOUT,99999) X, Y, IFAIL
         GO TO 20
      ELSE
         WRITE (NOUT,99998) IFAIL
      END IF
   40 CONTINUE
*
99999 FORMAT (1X,1P,2E12.3,I7)
99998 FORMAT (1X,' ** S11AAF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

```
S11AAF Example Program Data
            -0.5
             0.0
             0.5
            -0.9999
             3.0
```

9.3 Program Results

```
 S11AAF Example Program Results

      X           Y        IFAIL

   -5.000E-01  -5.493E-01      0
    0.000E+00   0.000E+00      0
    5.000E-01   5.493E-01      0
   -9.999E-01  -4.952E+00      0
    3.000E+00   0.000E+00      1
```