

NAG Library Routine Document

S11ABF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

S11ABF returns the value of the inverse hyperbolic sine, $\operatorname{arcsinh} x$, via the routine name.

2 Specification

double precision FUNCTION S11ABF(X, IFAIL)
 INTEGER IFAIL
double precision X

3 Description

S11ABF calculates an approximate value for the inverse hyperbolic sine of its argument, $\operatorname{arcsinh} x$.

For $|x| \leq 1$ it is based on the Chebyshev expansion

$$\operatorname{arcsinh} x = x \times y(t) = x \sum_{r=0}^{\prime} c_r T_r(t), \quad \text{where } t = 2x^2 - 1.$$

For $|x| > 1$ it uses the fact that

$$\operatorname{arcsinh} x = \operatorname{sign} x \times \ln\left(|x| + \sqrt{x^2 + 1}\right).$$

This form is used directly for $1 < |x| < 10^k$, where $k = n/2 + 1$, and the machine uses approximately n decimal place arithmetic.

For $|x| \geq 10^k$, $\sqrt{x^2 + 1}$ is equal to $|x|$ to within the accuracy of the machine and hence we can guard against premature overflow and, without loss of accuracy, calculate

$$\operatorname{arcsinh} x = \operatorname{sign} x \times (\ln 2 + \ln|x|).$$

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – *double precision* *Input*
On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1 . If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the

recommended value is 0. When the value -1 or 1 is used it is essential to test the value of **IFAIL** on exit.

6 Error Indicators and Warnings

None.

7 Accuracy

If δ and ϵ are the relative errors in the argument and the result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\sqrt{1+x^2} \operatorname{arcsinh} x} \delta \right|.$$

That is, the relative error in the argument, x , is amplified by a factor at least $\frac{x}{\sqrt{1+x^2} \operatorname{arcsinh} x}$, in the result.

The equality should hold if δ is greater than the *machine precision* (δ due to data errors etc.) but if δ is simply due to round-off in the machine representation it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the amplification factor is shown in the following graph:

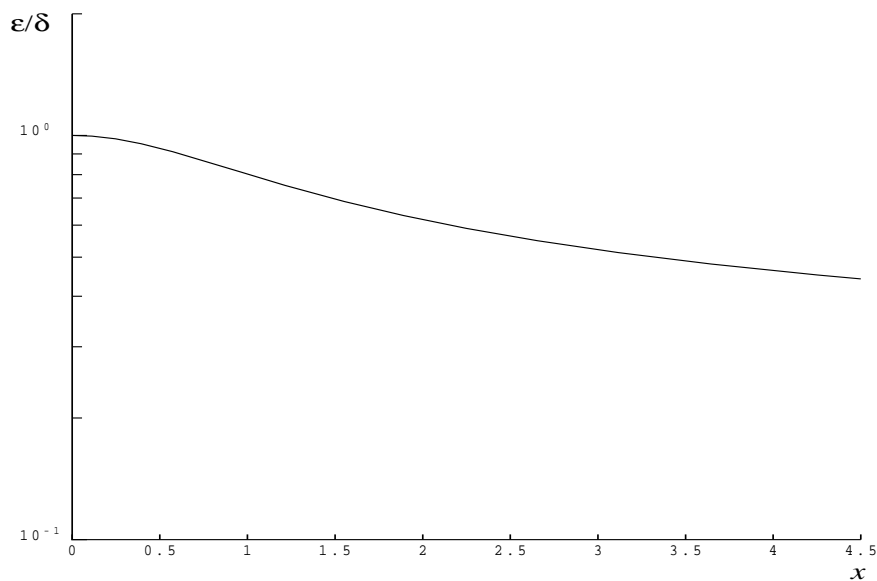


Figure 1

It should be noted that this factor is always less than or equal to one. For large x we have the absolute error in the result, E , in principle, given by

$$E \sim \delta.$$

This means that eventually accuracy is limited by *machine precision*.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*      S11ABF Example Program Text
*      Mark 14 Revised. NAG Copyright 1989.
*      .. Parameters ..
INTEGER          NIN, NOUT
PARAMETER       (NIN=5,NOUT=6)
*      .. Local Scalars ..
DOUBLE PRECISION X, Y
INTEGER         IFAIL
*      .. External Functions ..
DOUBLE PRECISION S11ABF
EXTERNAL        S11ABF
*      .. Executable Statements ..
WRITE (NOUT,*) 'S11ABF Example Program Results'
*      Skip heading in data file
READ (NIN,*)
WRITE (NOUT,*)
WRITE (NOUT,*) '      X          Y          IFAIL'
WRITE (NOUT,*)
20 READ (NIN,*,END=40) X
   IFAIL = 1
*
   Y = S11ABF(X,IFAIL)
*
   IF (IFAIL.GE.0) THEN
       WRITE (NOUT,99999) X, Y, IFAIL
       GO TO 20
   ELSE
       WRITE (NOUT,99998) IFAIL
   END IF
40 CONTINUE
*
99999 FORMAT (1X,1P,2E12.3,I7)
99998 FORMAT (1X,' ** S11ABF returned with IFAIL = ',I5)
END
```

9.2 Program Data

```
S11ABF Example Program Data
      -2.0
      -0.5
      1.0
      6.0
```

9.3 Program Results

```
S11ABF Example Program Results
```

X	Y	IFAIL
-2.000E+00	-1.444E+00	0
-5.000E-01	-4.812E-01	0
1.000E+00	8.814E-01	0
6.000E+00	2.492E+00	0
