NAG Library Routine Document

S11ACF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S11ACF returns the value of the inverse hyperbolic cosine, $\operatorname{arccosh} x$, via the routine name. The result is in the principal positive branch.

2 Specification

double precision `FUNCTION S11ACF(X, IFAIL)`

```
INTEGER                  IFAIL
double precision         X
```

3 Description

S11ACF calculates an approximate value for the inverse hyperbolic cosine, $\operatorname{arccosh} x$. It is based on the relation

$$\operatorname{arccosh} x = \ln\left(x + \sqrt{x^2 - 1}\right).$$

This form is used directly for $1 < x < 10^k$, where $k = n/2 + 1$, and the machine uses approximately n decimal place arithmetic.

For $x \geq 10^k$, $\sqrt{x^2 - 1}$ is equal to $\sqrt{x}$ to within the accuracy of the machine and hence we can guard against premature overflow and, without loss of accuracy, calculate

$$\operatorname{arccosh} x = \ln 2 + \ln x.$$

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – ***double precision*** *Input*

On entry: the argument x of the function.

Constraint: X $\geq$ 1.0.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

The routine has been called with an argument less than 1.0, for which $\operatorname{arccosh} x$ is not defined. The result returned is zero.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\sqrt{x^2-1}\operatorname{arccosh} x} \times \delta \right|.$$

That is the relative error in the argument is amplified by a factor at least $\dfrac{x}{\sqrt{x^2-1}\operatorname{arccosh} x}$ in the result. The equality should apply if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is simply a result of round-off in the machine representation it is possible that an extra figure may be lost in internal calculation and round-off. The behaviour of the amplification factor is shown in the following graph:

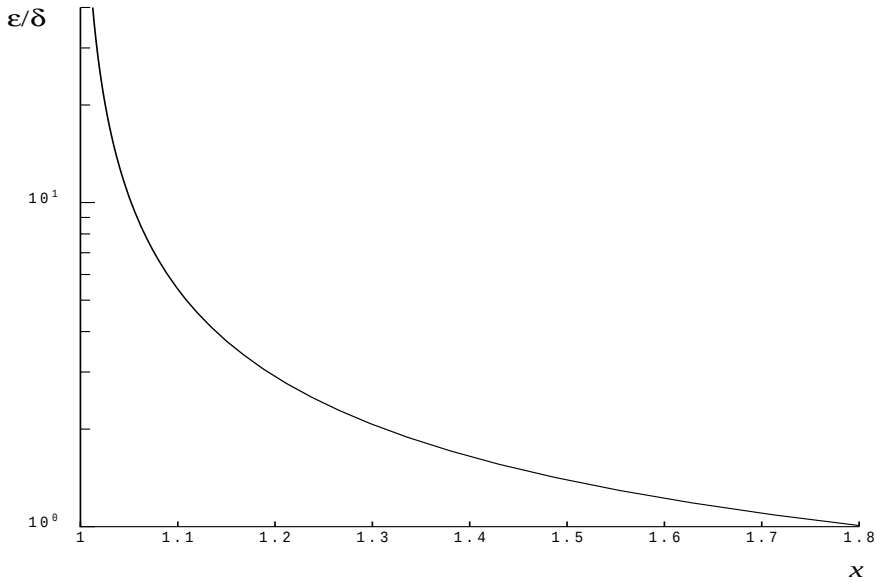

Figure 1

It should be noted that for $x > 2$ the factor is always less than 1.0. For large x we have the absolute error E in the result, in principle, given by

$$E \sim \delta.$$

This means that eventually accuracy is limited by ***machine precision***. More significantly for x close to 1, $x - 1 \sim \delta$, the above analysis becomes inapplicable due to the fact that both function and argument are bounded, $x \geq 1$, $\operatorname{arccosh} x \geq 0$. In this region we have

$$E \sim \sqrt{\delta}.$$

That is, there will be approximately half as many decimal places correct in the result as there were correct figures in the argument.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*     S11ACF Example Program Text
*     Mark 14 Revised. NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*     .. External Functions ..
      DOUBLE PRECISION S11ACF
      EXTERNAL         S11ACF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S11ACF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X            Y        IFAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S11ACF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
         WRITE (NOUT,99999) X, Y, IFAIL
         GO TO 20
      ELSE
         WRITE (NOUT,99998) IFAIL
      END IF
   40 CONTINUE
*
99999 FORMAT (1X,1P,2E12.3,I7)
99998 FORMAT (1X,' ** S11ACF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

```
S11ACF Example Program Data
             1.00
             2.0
             5.0
            10.0
            -0.5
```

9.3 Program Results

```
 S11ACF Example Program Results

      X            Y        IFAIL

    1.000E+00   0.000E+00      0
    2.000E+00   1.317E+00      0
    5.000E+00   2.292E+00      0
    1.000E+01   2.993E+00      0
   -5.000E-01   0.000E+00      1
```