NAG Library Routine Document

S13ADF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S13ADF returns the value of the sine integral

$$\mathrm{Si}(x) = \int_0^x \frac{\sin u}{u}\,du,$$

via the routine name.

2 Specification

double precision `FUNCTION S13ADF(X, IFAIL)`

`INTEGER` `IFAIL`
double precision `X`

3 Description

S13ADF calculates an approximate value for $\mathrm{Si}(x)$.

For $|x| \leq 16.0$ it is based on the Chebyshev expansion

$$\mathrm{Si}(x) = x{\sum_{r=0}}' a_r T_r(t), t = 2\left(\frac{x}{16}\right)^2 - 1.$$

For $16 < |x| < x_{\mathrm{hi}}$, where x_{hi} is an implementation-dependent number,

$$\mathrm{Si}(x) = \mathrm{sign}(x)\left\{\frac{\pi}{2} - \frac{f(x)\cos x}{x} - \frac{g(x)\sin x}{x^2}\right\}$$

where $f(x) = {\sum_{r=0}}' f_r T_r(t)$ and $g(x) = {\sum_{r=0}}' g_r T_r(t)$, $t = 2\left(\frac{16}{x}\right)^2 - 1$.

For $|x| \geq x_{\mathrm{hi}}$, $\mathrm{Si}(x) = \frac{1}{2}\pi \,\mathrm{sign}\, x$ to within ***machine precision***.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – ***double precision*** *Input*

On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

There are no failure exits from S13ADF. The parameter IFAIL has been included for consistency with other routines in this chapter.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left|\frac{\delta \sin x}{\mathrm{Si}(x)}\right|.$$

The equality may hold if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is simply due to round-off in the machine representation, then since the factor relating δ to ϵ is always less than one, the accuracy will be limited by ***machine precision***.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*     S13ADF Example Program Text
*     Mark 14 Revised. NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*     .. External Functions ..
      DOUBLE PRECISION S13ADF
      EXTERNAL         S13ADF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S13ADF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X           Y        IFAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S13ADF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
         WRITE (NOUT,99999) X, Y, IFAIL
         GO TO 20
```

```
      ELSE
         WRITE (NOUT,99998) IFAIL
      END IF
   40 CONTINUE
*
99999 FORMAT (1X,1P,2E12.3,I7)
99998 FORMAT (1X,' ** S13ADF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

```
S13ADF Example Program Data
             0.0
             0.2
             0.4
             0.6
             0.8
             1.0
```

9.3 Program Results

```
 S13ADF Example Program Results

      X           Y        IFAIL

    0.000E+00   0.000E+00      0
    2.000E-01   1.996E-01      0
    4.000E-01   3.965E-01      0
    6.000E-01   5.881E-01      0
    8.000E-01   7.721E-01      0
    1.000E+00   9.461E-01      0
```