# NAG Library Routine Document

# S14ACF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

## 1 Purpose

S14ACF returns a value of the function $\psi(x) - \ln x$, where $\psi$ is the psi function $\psi(x) = \dfrac{d}{dx}\ln\Gamma(x) = \dfrac{\Gamma'(x)}{\Gamma(x)}$.

## 2 Specification

***double precision*** `FUNCTION S14ACF(X, IFAIL)`

```
INTEGER                  IFAIL
double precision         X
```

## 3 Description

S14ACF returns a value of the function $\psi(x) - \ln x$. The psi function is computed without the logarithmic term so that when $x$ is large, sums or differences of psi functions may be computed without unnecessary loss of precision, by analytically combining the logarithmic terms. For example, the difference $d = \psi\left(x + \frac{1}{2}\right) - \psi(x)$ has an asymptotic behaviour for large $x$ given by $d \sim \ln\left(x + \frac{1}{2}\right) - \ln x + O\left(\dfrac{1}{x^2}\right) \sim \ln\left(1 + \dfrac{1}{2x}\right) \sim \dfrac{1}{2x}$.

Computing $d$ directly would amount to subtracting two large numbers which are close to $\ln\left(x + \frac{1}{2}\right)$ and $\ln x$ to produce a small number close to $\frac{1}{2x}$, resulting in a loss of significant digits. However, using this routine to compute $f(x) = \psi(x) - \ln x$, we can compute $d = f\left(x + \frac{1}{2}\right) - f(x) + \ln\left(1 + \frac{1}{2x}\right)$, and the dominant logarithmic term may be computed accurately from its power series when $x$ is large. Thus we avoid the unnecessary loss of precision.

The routine is derived from the routine PSIFN in Amos (1983).

## 4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Amos D E (1983) Algorithm 610: A portable FORTRAN subroutine for derivatives of the psi function *ACM Trans. Math. Software* **9** 494–502

## 5 Parameters

1: X – ***double precision*** *Input*

*On entry*: the argument $x$ of the function.

*Constraint*: X $> 0.0$.

2: IFAIL – INTEGER *Input/Output*

*On entry*: IFAIL must be set to $0$, $-1$ or $1$. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

*On exit*: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value $-1$ or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value $-1$ or 1 is used it is essential to test the value of IFAIL on exit.**

## 6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or $-1$, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, X $\leq 0.0$. S14ACF returns the value zero.

IFAIL $= 2$

No result is computed because underflow is likely. The value of X is too large. S14ACF returns the value zero.

IFAIL $= 3$

No result is computed because overflow is likely. The value of X is too small. S14ACF returns the value zero.

## 7 Accuracy

All constants in S14ACF are given to approximately 18 digits of precision. Calling the number of digits of precision in the floating-point arithmetic being used $t$, then clearly the maximum number of correct digits in the results obtained is limited by $p = \min(t, 18)$.

With the above proviso, results returned by this routine should be accurate almost to full precision, except at points close to the zero of $\psi(x)$, $x \simeq 1.461632$, where only absolute rather than relative accuracy can be obtained.

## 8 Further Comments

None.

## 9 Example

The example program reads values of the argument $x$ from a file, evaluates the function at each value of $x$ and prints the results.

### 9.1 Program Text

```
*     S14ACF Example Program Text
*     Mark 14 Release. NAG Copyright 1989
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      DOUBLE PRECISION F, X
      INTEGER          IFAIL
*     .. External Functions ..
      DOUBLE PRECISION S14ACF
      EXTERNAL         S14ACF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S14ACF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
```

```
      WRITE (NOUT,*)
      WRITE (NOUT,*) '       X        psi(X)-log(X)'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
      IFAIL = 1
*
      F = S14ACF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
         WRITE (NOUT,99999) X, F
         GO TO 20
      ELSE
         WRITE (NOUT,99998) IFAIL
      END IF
   40 CONTINUE
*
99999 FORMAT (1X,F12.4,F15.4)
99998 FORMAT (1X,' ** S14ACF returned with IFAIL = ',I5)
      END
```

## 9.2 Program Data

```
S14ACF Example Program Data
0.1
0.5
3.6
8.0
```

## 9.3 Program Results

```
 S14ACF Example Program Results

        X        psi(X)-log(X)

       0.1000        -8.1212
       0.5000        -1.2704
       3.6000        -0.1453
       8.0000        -0.0638
```