NAG Library Routine Document

S15ABF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S15ABF returns the value of the cumulative Normal distribution function, $P(x)$, via the routine name.

2 Specification

double precision `FUNCTION S15ABF(X, IFAIL)`

```
INTEGER                  IFAIL
double precision         X
```

3 Description

S15ABF evaluates an approximate value for the cumulative Normal distribution function

$$P(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2}\, du.$$

The routine is based on the fact that

$$P(x) = \tfrac{1}{2}\mathrm{erfc}\left(\frac{-x}{\sqrt{2}}\right)$$

and it calls S15ADF to obtain a value of *erfc* for the appropriate argument.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – ***double precision*** *Input*

On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

There are no failure exits from this routine. The parameter IFAIL is included for consistency with other routines in this chapter.

7 Accuracy

Because of its close relationship with *erfc*, the accuracy of this routine is very similar to that in S15ADF. If ϵ and δ are the relative errors in result and argument, respectively, they are in principle related by

$$|\epsilon| \simeq \left| \frac{xe^{-\frac{1}{2}x^2}}{\sqrt{2\pi}P(x)}\delta \right|$$

so that the relative error in the argument, x, is amplified by a factor, $\frac{xe^{-\frac{1}{2}x^2}}{\sqrt{2\pi}P(x)}$, in the result.

For x small and for x positive this factor is always less than one and accuracy is mainly limited by ***machine precision***.

For large negative x the factor behaves like $\sim x^2$ and hence to a certain extent relative accuracy is unavoidably lost.

However the absolute error in the result, E, is given by

$$|E| \simeq \left| \frac{xe^{-\frac{1}{2}x^2}}{\sqrt{2\pi}}\delta \right|$$

so absolute accuracy can be guaranteed for all x.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*     S15ABF Example Program Text
*     Mark 14 Revised. NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*     .. External Functions ..
      DOUBLE PRECISION S15ABF
      EXTERNAL         S15ABF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S15ABF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X           Y'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S15ABF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
         WRITE (NOUT,99999) X, Y
         GO TO 20
      ELSE
         WRITE (NOUT,99998) IFAIL
```

```
      END IF
   40 CONTINUE
*
99999 FORMAT (1X,1P,2E12.3)
99998 FORMAT (1X,' ** S15ABF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

```
S15ABF Example Program Data
          -20.0
           -1.0
            0.0
            1.0
            2.0
           20.0
```

9.3 Program Results

```
 S15ABF Example Program Results

      X           Y

   -2.000E+01   2.754E-89
   -1.000E+00   1.587E-01
    0.000E+00   5.000E-01
    1.000E+00   8.413E-01
    2.000E+00   9.772E-01
    2.000E+01   1.000E+00
```