

NAG Library Routine Document

S30SAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

S30SAF computes the Asian geometric continuous average-rate option price.

2 Specification

```
SUBROUTINE S30SAF (CALPUT, M, N, X, S, T, SIGMA, R, B, P, LDP, IFAIL)
INTEGER          M, N, LDP, IFAIL
REAL (KIND=nag_wp) X(M), S, T(N), SIGMA, R, B, P(LDP,N)
CHARACTER(1)    CALPUT
```

3 Description

S30SAF computes the price of an Asian geometric continuous average-rate option for constant volatility, σ , risk-free rate, r , and cost of carry, b (see Kemna and Vorst (1990)). For a given strike price, X , the price of a call option with underlying price, S , and time to expiry, T , is

$$P_{\text{call}} = Se^{(\bar{b}-r)T}\Phi(\bar{d}_1) - Xe^{-rT}\Phi(\bar{d}_2),$$

and the corresponding put option price is

$$P_{\text{put}} = Xe^{-rT}\Phi(-\bar{d}_2) - Se^{(\bar{b}-r)T}\Phi(-\bar{d}_1),$$

where

$$\bar{d}_1 = \frac{\ln(S/X) + (\bar{b} + \sigma^2/2)T}{\sigma\sqrt{T}}$$

and

$$\bar{d}_2 = \bar{d}_1 - \sigma\sqrt{T},$$

with

$$\bar{\sigma} = \frac{\sigma}{\sqrt{3}}, \quad \bar{b} = \frac{1}{2}\left(b - \frac{\sigma^2}{6}\right).$$

Φ is the cumulative Normal distribution function,

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x \exp(-y^2/2) dy.$$

4 References

Kemna A and Vorst A (1990) A pricing method for options based on average asset values *Journal of Banking and Finance* **14** 113–129

5 Parameters

- 1: CALPUT – CHARACTER(1) *Input*
On entry: determines whether the option is a call or a put.
 CALPUT = 'C'
 A call. The holder has a right to buy.
 CALPUT = 'P'
 A put. The holder has a right to sell.
Constraint: CALPUT = 'C' or 'P'.
- 2: M – INTEGER *Input*
On entry: the number of strike prices to be used.
Constraint: $M \geq 1$.
- 3: N – INTEGER *Input*
On entry: the number of times to expiry to be used.
Constraint: $N \geq 1$.
- 4: X(M) – REAL (KIND=nag_wp) array *Input*
On entry: X(*i*) must contain X_i , the *i*th strike price, for $i = 1, 2, \dots, M$.
Constraint: $X(i) \geq z$ and $X(i) \leq 1/z$, where $z = X02AMF()$, the safe range parameter, for $i = 1, 2, \dots, M$.
- 5: S – REAL (KIND=nag_wp) *Input*
On entry: S, the price of the underlying asset.
Constraint: $S \geq z$ and $S \leq 1.0/z$, where $z = X02AMF()$, the safe range parameter.
- 6: T(N) – REAL (KIND=nag_wp) array *Input*
On entry: T(*i*) must contain T_i , the *i*th time, in years, to expiry, for $i = 1, 2, \dots, N$.
Constraint: $T(i) \geq z$, where $z = X02AMF()$, the safe range parameter, for $i = 1, 2, \dots, N$.
- 7: SIGMA – REAL (KIND=nag_wp) *Input*
On entry: σ , the volatility of the underlying asset. Note that a rate of 15% should be entered as 0.15.
Constraint: SIGMA > 0.0.
- 8: R – REAL (KIND=nag_wp) *Input*
On entry: r , the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.
Constraint: R \geq 0.0.
- 9: B – REAL (KIND=nag_wp) *Input*
On entry: b , the annual cost of carry rate. Note that a rate of 8% should be entered as 0.08.
- 10: P(LDP,N) – REAL (KIND=nag_wp) array *Output*
On exit: the leading $M \times N$ part of the array P contains the computed option prices.

- 11: LDP – INTEGER *Input*
On entry: the first dimension of the array P as declared in the (sub)program from which S30SAF is called.
Constraint: $LDP \geq M$.
- 12: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.
 For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**
On exit: IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, CALPUT \neq 'C' or 'P'.

IFAIL = 2

On entry, $M \leq 0$.

IFAIL = 3

On entry, $N \leq 0$.

IFAIL = 4

On entry, $X(i) < z$ or $X(i) > 1/z$, where $z = X02AMF()$, the safe range parameter.

IFAIL = 5

On entry, $S < z$ or $S > 1.0/z$, where $z = X02AMF()$, the safe range parameter.

IFAIL = 6

On entry, $T(i) < z$, where $z = X02AMF()$, the safe range parameter.

IFAIL = 7

On entry, $SIGMA \leq 0.0$.

IFAIL = 8

On entry, $R < 0.0$.

IFAIL = 11

On entry, $LDP < M$.

7 Accuracy

The accuracy of the output is dependent on the accuracy of the cumulative Normal distribution function, Φ . This is evaluated using a rational Chebyshev expansion, chosen so that the maximum relative error in the expansion is of the order of the *machine precision* (see S15ABF and S15ADF). An accuracy close to *machine precision* can generally be expected.

8 Further Comments

None.

9 Example

This example computes the price of an Asian geometric continuous average-rate put with a time to expiry of 3 months, a stock price of 80 and a strike price of 85. The risk-free interest rate is 5% per year, the cost of carry is 8% and the volatility is 20% per year.

9.1 Program Text

```

Program s30safe

!      S30SAF Example Program Text

!      Mark 24 Release. NAG Copyright 2012.

!      .. Use Statements ..
      Use nag_library, Only: nag_wp, s30saf
!      .. Implicit None Statement ..
      Implicit None
!      .. Parameters ..
      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
      Real (Kind=nag_wp)          :: b, r, s, sigma
      Integer                     :: i, ifail, j, ldp, m, n
      Character (1)               :: calput
!      .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable :: p(:,,:), t(:,), x(:)
!      .. Executable Statements ..
      Write (nout,*) 'S30SAF Example Program Results'

!      Skip heading in data file
      Read (nin,*)

      Read (nin,*) calput
      Read (nin,*) s, sigma, r, b
      Read (nin,*) m, n

      ldp = m
      Allocate (p(ldp,n),t(n),x(m))

      Read (nin,*)(x(i),i=1,m)
      Read (nin,*)(t(i),i=1,n)

      ifail = 0
      Call s30saf(calput,m,n,x,s,t,sigma,r,b,p,ldp,ifail)

      Write (nout,*)
      Write (nout,*) 'Asian Option: Geometric Continuous Average-Rate'

      Select Case (calput)
      Case ('C','c')
         Write (nout,*) 'Asian Call :'
      Case ('P','p')
         Write (nout,*) 'Asian Put :'
      End Select

```

```

Write (nout,99998) ' Spot           = ', s
Write (nout,99998) ' Volatility    = ', sigma
Write (nout,99998) ' Rate          = ', r
Write (nout,99998) ' Cost of carry = ', b

Write (nout,*)
Write (nout,*) ' Strike    Expiry    Option Price'

Do i = 1, m

  Do j = 1, n
    Write (nout,99999) x(i), t(j), p(i,j)
  End Do

End Do

99999 Format (1X,2(F9.4,1X),6X,F9.4)
99998 Format (A,1X,F8.4)
End Program s30safe

```

9.2 Program Data

```

S30SAF Example Program Data
'P'           : Call = 'C', Put = 'P'
80.0 0.2 0.05 0.08 : S, SIGMA, R, B
1 1           : M, N
85.0         : X(I), I = 1,2,...M
0.25        : T(I), I = 1,2,...N

```

9.3 Program Results

S30SAF Example Program Results

Asian Option: Geometric Continuous Average-Rate

Asian Put :

```

Spot           = 80.0000
Volatility     = 0.2000
Rate           = 0.0500
Cost of carry  = 0.0800

```

```

Strike    Expiry    Option Price
85.0000   0.2500         4.6922

```
