# NAG Library Routine Document

# F06TSF

## 1 Purpose

F06TSF performs a $QR$ or $RQ$ factorization (as a sequence of plane rotations) of a complex upper spiked matrix.

## 2 Specification

```
SUBROUTINE F06TSF (SIDE, N, K1, K2, C, S, A, LDA)

INTEGER               N, K1, K2, LDA
REAL (KIND=nag_wp)    C(K2-1)
COMPLEX (KIND=nag_wp) S(*), A(LDA,*)
CHARACTER(1)          SIDE
```

## 3 Description

F06TSF transforms an $n$ by $n$ complex upper spiked matrix $H$ to upper triangular form $R$ by applying a complex unitary matrix $P$ from the left or the right. $H$ is assumed to have real diagonal elements except where the spike joins the diagonal; $R$ has real diagonal elements. $P$ is formed as a sequence of plane rotations in planes $k_1$ to $k_2$.

If SIDE = 'L', $H$ is assumed to have a row spike, with nonzero elements $h_{k_2,k}$, for $k = k_1, \ldots, k_2 - 1$. The rotations are applied from the left:

$$PH = R,$$

where $P = DP_{k_2-1} \cdots P_{k_1+1}P_{k_1}$, $P_k$ is a rotation in the $(k, k_2)$ plane and $D = \text{diag}(1, \ldots, 1, d_{k_2}, 1, \ldots, 1)$ with $|d_{k_2}| = 1$.

If SIDE = 'R', $H$ is assumed to have a column spike, with nonzero elements $h_{k+1,k_1}$, for $k = k_1, \ldots, k_2 - 1$. The rotations are applied from the right:

$$HP^{\text{H}} = R,$$

where $P = DP_{k_1}P_{k_1+1} \cdots P_{k_2-1}$, $P_k$ is a rotation in the $(k_1, k+1)$ plane and $D = \text{diag}(1, \ldots, 1, d_{k_1}, 1, \ldots, 1)$ with $|d_{k_1}| = 1$.

The 2 by 2 plane rotation part of $P_k$ has the form

$$\begin{pmatrix} c_k & \bar{s}_k \\ -s_k & c_k \end{pmatrix}$$

with $c_k$ real.

## 4 References

None.

## 5 Parameters

1: SIDE – CHARACTER(1) *Input*

*On entry*: specifies whether $H$ is operated on from the left or the right.

SIDE = 'L'
  $H$ is pre-multiplied from the left.

SIDE = 'R'
  $H$ is post-multiplied from the right.

*Constraint*: SIDE = 'L' or 'R'.

2: N – INTEGER *Input*

*On entry*: $n$, the order of the matrix $H$.

*Constraint*: $N \geq 0$.

3: K1 – INTEGER *Input*
4: K2 – INTEGER *Input*

*On entry*: the values $k_1$ and $k_2$.

If $K1 < 1$ or $K2 \leq K1$ or $K2 > N$, an immediate return is effected.

5: C(K2 − 1) – REAL (KIND=nag_wp) array *Output*

*On exit*: C($k$) holds $c_k$, the cosine of the rotation $P_k$, for $k = k_1, \ldots, k_2 - 1$.

6: S(∗) – COMPLEX (KIND=nag_wp) array *Input/Output*

**Note**: the dimension of the array S must be at least $K2 - K1$.

*On entry*: the nonzero elements of the spike of $H$: S($k$) must hold $h_{k_2,k}$ if SIDE = 'L', and $h_{k+1,k_1}$ if SIDE = 'R', for $k = k_1, \ldots, k_2 - 1$.

*On exit*: S($k$) holds $s_k$, the sine of the rotation $P_k$, for $k = k_1, \ldots, k_2 - 1$; S($k_2$) holds $d_{k_2}$, the $k_2$th diagonal element of $D$, if SIDE = 'L', or $d_{k_1}$, the $k_1$th diagonal element of $D$, if SIDE = 'R'.

7: A(LDA, ∗) – COMPLEX (KIND=nag_wp) array *Input/Output*

**Note**: the second dimension of the array A must be at least N.

*On entry*: the upper triangular part of the $n$ by $n$ upper spiked matrix $H$. The imaginary parts of the diagonal elements must be zero, except for the $(k_2, k_2)$ element if SIDE = 'L', or the $(k_1, k_1)$ element if SIDE = 'R'.

*On exit*: the upper triangular matrix $R$. The imaginary parts of the diagonal elements are set to zero.

8: LDA – INTEGER *Input*

*On entry*: the first dimension of the array A as declared in the (sub)program from which F06TSF is called.

*Constraint*: $LDA \geq \max(1, N)$.

## 6 Error Indicators and Warnings

None.

## 7 Accuracy

Not applicable.

## 8    Parallelism and Performance

F06TSF is not threaded by NAG in any implementation.

F06TSF makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9    Further Comments

None.

## 10    Example

None.