NAG Library Routine Document

F07JRF (ZPTTRF)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07JRF (ZPTTRF) computes the modified Cholesky factorization of a complex n by n Hermitian positive definite tridiagonal matrix A.

2 Specification

```
SUBROUTINE F07JRF (N, D, E, INFO)
INTEGER                 N, INFO
REAL (KIND=nag_wp)      D(*)
COMPLEX (KIND=nag_wp) E(*)
```

The routine may be called by its LAPACK name ***zpttrf***.

3 Description

F07JRF (ZPTTRF) factorizes the matrix A as

$$A = LDL^{\mathrm{H}},$$

where L is a unit lower bidiagonal matrix and D is a diagonal matrix with positive diagonal elements. The factorization may also be regarded as having the form $U^{\mathrm{H}}DU$, where U is a unit upper bidiagonal matrix.

4 References

None.

5 Arguments

1: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

2: D($*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the dimension of the array D must be at least $\max(1, \mathrm{N})$.

On entry: must contain the n diagonal elements of the matrix A.

On exit: is overwritten by the n diagonal elements of the diagonal matrix D from the LDL^{H} factorization of A.

3: E($*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the dimension of the array E must be at least $\max(1, \mathrm{N} - 1)$.

On entry: must contain the $(n-1)$ subdiagonal elements of the matrix A.

On exit: is overwritten by the $(n-1)$ subdiagonal elements of the lower bidiagonal matrix L. (E can also be regarded as containing the $(n-1)$ superdiagonal elements of the upper bidiagonal matrix U.)

4: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0 and INFO $<$ N

The leading minor of order $\langle value \rangle$ is not positive definite, the factorization could not be completed.

INFO > 0 and INFO $=$ N

The leading minor of order n is not positive definite, the factorization was completed, but D(N) ≤ 0.

7 Accuracy

The computed factorization satisfies an equation of the form

$$A + E = LDL^{\mathrm{H}},$$

where

$$\|E\|_\infty = O(\epsilon)\|A\|_\infty$$

and ϵ is the ***machine precision***.

Following the use of this routine, F07JSF (ZPTTRS) can be used to solve systems of equations $AX = B$, and F07JUF (ZPTCON) can be used to estimate the condition number of A.

8 Parallelism and Performance

F07JRF (ZPTTRF) is not threaded in any implementation.

9 Further Comments

The total number of floating-point operations required to factorize the matrix A is proportional to n.

The real analogue of this routine is F07JDF (DPTTRF).

10 Example

This example factorizes the Hermitian positive definite tridiagonal matrix A given by

$$A = \begin{pmatrix} 16.0 & 16.0 - 16.0i & 0 & 0 \\ 16.0 + 16.0i & 41.0 & 18.0 + 9.0i & 0 \\ 0 & 18.0 - 9.0i & 46.0 & 1.0 + 4.0i \\ 0 & 0 & 1.0 - 4.0i & 21.0 \end{pmatrix}.$$

10.1 Program Text

```
    Program f07jrfe

!     F07JRF Example Program Text

!     Mark 26 Release. NAG Copyright 2016.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, zpttrf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                          :: info, n
!     .. Local Arrays ..
      Complex (Kind=nag_wp), Allocatable :: e(:)
      Real (Kind=nag_wp), Allocatable  :: d(:)
!     .. Executable Statements ..
      Write (nout,*) 'F07JRF Example Program Results'
      Write (nout,*)
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n

      Allocate (e(n-1),d(n))

!     Read the lower bidiagonal part of the tridiagonal matrix A from
!     data file

      Read (nin,*) d(1:n)
      Read (nin,*) e(1:n-1)

!     Factorize the tridiagonal matrix A
!     The NAG name equivalent of zpttrf is f07jrf
      Call zpttrf(n,d,e,info)

      If (info>0) Then
        Write (nout,99999) 'The leading minor of order ', info,                &
          ' is not positive definite'
      End If

!     Print details of the factorization

      Write (nout,*) 'Details of factorization'
      Write (nout,*)
      Write (nout,*) ' The diagonal elements of D'
      Write (nout,99998) d(1:n)
      Write (nout,*)
      Write (nout,*) ' Subdiagonal elements of the Cholesky factor L'
      Write (nout,99998) e(1:n-1)

99999 Format (1X,A,I3,A)
99998 Format (1X,8F9.4)
    End Program f07jrfe
```

10.2 Program Data

```
F07JRF Example Program Data
    4                                                    :Value of N
   16.0          41.0          46.0          21.0        :End of diagonal D
 ( 16.0, 16.0) ( 18.0, -9.0) (  1.0, -4.0)               :End of sub-diagonal E
```

10.3 Program Results

```
 F07JRF Example Program Results

 Details of factorization

  The diagonal elements of D
   16.0000   9.0000   1.0000   4.0000

  Subdiagonal elements of the Cholesky factor L
    1.0000   1.0000   2.0000  -1.0000   1.0000  -4.0000
```