

NAG Library Routine Document

F08YTF (ZTGEXC)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F08YTF (ZTGEXC) reorders the generalized Schur factorization of a complex matrix pair in generalized Schur form.

2 Specification

SUBROUTINE F08YTF (WANTQ, WANTZ, N, A, LDA, B, LDB, Q, LDQ, Z, LDZ, &
IFST, ILST, INFO)

INTEGER N, LDA, LDB, LDQ, LDZ, IFST, ILST, INFO
COMPLEX (KIND=nag_wp) A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)
LOGICAL WANTQ, WANTZ

The routine may be called by its LAPACK name *ztgexc*.

3 Description

F08YTF (ZTGEXC) reorders the generalized complex n by n matrix pair (S, T) in generalized Schur form, so that the diagonal element of (S, T) with row index i_1 is moved to row i_2 , using a unitary equivalence transformation. That is, S and T are factorized as

$$S = \hat{Q}\hat{S}\hat{Z}^H, \quad T = \hat{Q}\hat{T}\hat{Z}^H,$$

where (\hat{S}, \hat{T}) are also in generalized Schur form.

The pair (S, T) are in generalized Schur form if S and T are upper triangular as returned, for example, by F08XNF (ZGGES), or F08XSF (ZHGEQZ) with JOB = 'S'.

If S and T are the result of a generalized Schur factorization of a matrix pair (A, B)

$$A = QSZ^H, \quad B = QTZ^H$$

then, optionally, the matrices Q and Z can be updated as $Q\hat{Q}$ and $Z\hat{Z}$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

5 Arguments

1: WANTQ – LOGICAL *Input*

On entry: if WANTQ = .TRUE., update the left transformation matrix Q .

If WANTQ = .FALSE., do not update Q .

2: WANTZ – LOGICAL *Input*

On entry: if WANTZ = .TRUE., update the right transformation matrix Z .

If WANTZ = .FALSE., do not update Z .

- 3: N – INTEGER *Input*
On entry: n , the order of the matrices S and T .
Constraint: $N \geq 0$.
- 4: A(LDA,*) – COMPLEX (KIND=nag_wp) array *Input/Output*
Note: the second dimension of the array A must be at least $\max(1, N)$.
On entry: the matrix S in the pair (S, T) .
On exit: the updated matrix \hat{S} .
- 5: LDA – INTEGER *Input*
On entry: the first dimension of the array A as declared in the (sub)program from which F08YTF (ZTGEXC) is called.
Constraint: $LDA \geq \max(1, N)$.
- 6: B(LDB,*) – COMPLEX (KIND=nag_wp) array *Input/Output*
Note: the second dimension of the array B must be at least $\max(1, N)$.
On entry: the matrix T , in the pair (S, T) .
On exit: the updated matrix \hat{T} .
- 7: LDB – INTEGER *Input*
On entry: the first dimension of the array B as declared in the (sub)program from which F08YTF (ZTGEXC) is called.
Constraint: $LDB \geq \max(1, N)$.
- 8: Q(LDQ,*) – COMPLEX (KIND=nag_wp) array *Input/Output*
Note: the second dimension of the array Q must be at least $\max(1, N)$ if WANTQ = .TRUE., and at least 1 otherwise.
On entry: if WANTQ = .TRUE., the unitary matrix Q .
On exit: if WANTQ = .TRUE., the updated matrix $Q\hat{Q}$.
 If WANTQ = .FALSE., Q is not referenced.
- 9: LDQ – INTEGER *Input*
On entry: the first dimension of the array Q as declared in the (sub)program from which F08YTF (ZTGEXC) is called.
Constraints:
 if WANTQ = .TRUE., $LDQ \geq \max(1, N)$;
 otherwise $LDQ \geq 1$.
- 10: Z(LDZ,*) – COMPLEX (KIND=nag_wp) array *Input/Output*
Note: the second dimension of the array Z must be at least $\max(1, N)$ if WANTZ = .TRUE., and at least 1 otherwise.
On entry: if WANTZ = .TRUE., the unitary matrix Z .
On exit: if WANTZ = .TRUE., the updated matrix $Z\hat{Z}$.
 If WANTZ = .FALSE., Z is not referenced.

11: LDZ – INTEGER *Input*

On entry: the first dimension of the array Z as declared in the (sub)program from which F08YTF (ZTGEXC) is called.

Constraints:

if WANTZ = .TRUE., LDZ \geq max(1, N);
otherwise LDZ \geq 1.

12: IFST – INTEGER *Input*

13: ILST – INTEGER *Input/Output*

On entry: the indices i_1 and i_2 that specify the reordering of the diagonal elements of (S, T) . The element with row index IFST is moved to row ILST, by a sequence of swapping between adjacent diagonal elements.

On exit: ILST points to the row in its final position.

Constraint: $1 \leq$ IFST \leq N and $1 \leq$ ILST \leq N.

14: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO = $-i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO = 1

The transformed matrix pair (\hat{S}, \hat{T}) would be too far from generalized Schur form; the problem is ill-conditioned. (S, T) may have been partially reordered, and ILST points to the first row of the current position of the block being moved.

7 Accuracy

The computed generalized Schur form is nearly the exact generalized Schur form for nearby matrices $(S + E)$ and $(T + F)$, where

$$\|E\|_2 = O\epsilon\|S\|_2 \quad \text{and} \quad \|F\|_2 = O\epsilon\|T\|_2,$$

and ϵ is the *machine precision*. See Section 4.11 of Anderson *et al.* (1999) for further details of error bounds for the generalized nonsymmetric eigenproblem.

8 Parallelism and Performance

F08YTF (ZTGEXC) is not threaded in any implementation.

9 Further Comments

The real analogue of this routine is F08YFF (DTGEXC).

10 Example

This example exchanges rows 4 and 1 of the matrix pair (S, T) , where

$$S = \begin{pmatrix} 4.0 + 4.0i & 1.0 + 1.0i & 1.0 + 1.0i & 2.0 - 1.0i \\ 0 & 2.0 + 1.0i & 1.0 + 1.0i & 1.0 + 1.0i \\ 0 & 0 & 2.0 - 1.0i & 1.0 + 1.0i \\ 0 & 0 & 0 & 6.0 - 2.0i \end{pmatrix}$$

and

$$T = \begin{pmatrix} 2.0 & 1.0 + 1.0i & 1.0 + 1.0i & 3.0 - 1.0i \\ 0 & 1.0 & 2.0 + 1.0i & 1.0 + 1.0i \\ 0 & 0 & 1.0 & 1.0 + 1.0i \\ 0 & 0 & 0 & 2.0 \end{pmatrix}.$$

10.1 Program Text

```

Program f08ytf

!      F08YTF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..
Use nag_library, Only: nag_wp, x04dbf, ztgexc
!      .. Implicit None Statement ..
Implicit None
!      .. Parameters ..
Integer, Parameter          :: nin = 5, nout = 6
Logical, Parameter         :: wantq = .False., wantz = .False.
!      .. Local Scalars ..
Integer                    :: i, ifail, ifst, ilst, info, lda,      &
                          ldb, ldq, ldz, n
!      .. Local Arrays ..
Complex (Kind=nag_wp), Allocatable :: a(:,,:), b(:,,:), q(:,,:), z(:,,:)
Character (1)                  :: clabs(1), rlabs(1)
!      .. Executable Statements ..
Write (nout,*) 'F08YTF Example Program Results'
Write (nout,*)
Flush (nout)
!      Skip heading in data file
Read (nin,*)
Read (nin,*) n
ldq = 1
ldz = 1
lda = n
ldb = n
Allocate (a(lda,n),b(ldb,n),q(ldq,1),z(ldz,1))

!      Read A and B from data file

Read (nin,*)(a(i,1:n),i=1,n)
Read (nin,*)(b(i,1:n),i=1,n)

!      Read the row indices

Read (nin,*) ifst, ilst

!      Reorder the A and B

!      The NAG name equivalent of ztgexc is f08ytf
Call ztgexc(wantq,wantz,n,a,lda,b,ldb,q,ldq,z,ldz,ifst,ilst,info)

If (info/=0) Then
  Write (nout,99999) info, ilst
  Write (nout,*)
  Flush (nout)
End If

```

```

!      Print reordered generalized Schur form

!      ifail: behaviour on error exit
!      =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
ifail = 0
Call x04dbf('General',' ',n,n,a,lda,'Bracketed','F7.4',      &
  'Reordered Schur matrix A','Integer',rlabs,'Integer',clabs,80,0,ifail)

Write (nout,*)
Flush (nout)

ifail = 0
Call x04dbf('General',' ',n,n,b,ldb,'Bracketed','F7.4',      &
  'Reordered Schur matrix B','Integer',rlabs,'Integer',clabs,80,0,ifail)

99999 Format (' Reordering could not be completed. INFO = ',I3,' ILST = ',I5)
End Program f08ytf

```

10.2 Program Data

F08YTF Example Program Data

```

4                                     :Value of N
( 4.0, 4.0) ( 1.0, 1.0) ( 1.0, 1.0) ( 2.0,-1.0)
( 0.0, 0.0) ( 2.0, 1.0) ( 1.0, 1.0) ( 1.0, 1.0)
( 0.0, 0.0) ( 0.0, 0.0) ( 2.0,-1.0) ( 1.0, 1.0)
( 0.0, 0.0) ( 0.0, 0.0) ( 0.0, 0.0) ( 6.0,-2.0)      :End of matrix A
( 2.0, 0.0) ( 1.0, 1.0) ( 1.0, 1.0) ( 3.0,-1.0)
( 0.0, 0.0) ( 1.0, 0.0) ( 2.0, 1.0) ( 1.0, 1.0)
( 0.0, 0.0) ( 0.0, 0.0) ( 1.0, 0.0) ( 1.0, 1.0)
( 0.0, 0.0) ( 0.0, 0.0) ( 0.0, 0.0) ( 2.0, 0.0)      :End of matrix B
1 4                                     :Values of IFST and ILST

```

10.3 Program Results

F08YTF Example Program Results

Reordered Schur matrix A

```

1 2 3 4
1 ( 3.7081, 3.7081) (-2.0834,-0.5688) ( 2.6374, 1.0772) ( 0.2845, 0.7991)
2 ( 0.0000, 0.0000) ( 1.6097, 1.5656) (-0.0634, 1.9234) (-0.0301, 0.9720)
3 ( 0.0000, 0.0000) ( 0.0000, 0.0000) ( 4.7029,-2.1187) ( 1.1379,-3.1199)
4 ( 0.0000, 0.0000) ( 0.0000, 0.0000) ( 0.0000, 0.0000) ( 2.3085,-1.8289)

```

Reordered Schur matrix B

```

1 2 3 4
1 ( 2.2249, 0.7416) (-1.1631, 1.5347) ( 2.2608, 2.0851) ( 1.1094,-0.3205)
2 ( 0.0000, 0.0000) ( 0.3308, 0.9482) ( 0.3919, 1.8172) (-0.6305, 1.6053)
3 ( 0.0000, 0.0000) ( 0.0000, 0.0000) ( 1.6227,-0.1653) ( 0.9966,-0.9074)
4 ( 0.0000, 0.0000) ( 0.0000, 0.0000) ( 0.0000, 0.0000) ( 0.1199,-1.0343)

```
