

NAG Toolbox

nag_sum_chebyshev (c06dc)

1 Purpose

nag_sum_chebyshev (c06dc) evaluates a polynomial from its Chebyshev series representation at a set of points.

2 Syntax

```
[res, ifail] = nag_sum_chebyshev(x, xmin, xmax, c, s, 'lx', lx, 'n', n)
[res, ifail] = c06dc(x, xmin, xmax, c, s, 'lx', lx, 'n', n)
```

3 Description

nag_sum_chebyshev (c06dc) evaluates, at each point in a given set X , the sum of a Chebyshev series of one of three forms according to the value of the parameter s :

$$s = 1: \quad 0.5c_1 + \sum_{j=2}^n c_j T_{j-1}(\bar{x})$$

$$s = 2: \quad 0.5c_1 + \sum_{j=2}^n c_j T_{2j-2}(\bar{x})$$

$$s = 3: \quad \sum_{j=1}^n c_j T_{2j-1}(\bar{x})$$

where \bar{x} lies in the range $-1.0 \leq \bar{x} \leq 1.0$. Here $T_r(x)$ is the Chebyshev polynomial of order r in \bar{x} , defined by $\cos(ry)$ where $\cos y = \bar{x}$.

It is assumed that the independent variable \bar{x} in the interval $[-1.0, +1.0]$ was obtained from your original variable $x \in X$, a set of real numbers in the interval $[x_{\min}, x_{\max}]$, by the linear transformation

$$\bar{x} = \frac{2x - (x_{\max} + x_{\min})}{x_{\max} - x_{\min}}.$$

The method used is based upon a three-term recurrence relation; for details see Clenshaw (1962).

The coefficients c_j are normally generated by other functions, for example they may be those returned by the interpolation function nag_interp_1d_cheb (e01ae) (in vector \mathbf{a}), by a least squares fitting function in Chapter E02, or as the solution of a boundary value problem by nag_ode_bvp_coll_nth (d02ja), nag_ode_bvp_coll_sys (d02jb) or nag_ode_bvp_ps_lin_solve (d02ue).

4 References

Clenshaw C W (1962) Chebyshev Series for Mathematical Functions *Mathematical tables* HMSO

5 Parameters

5.1 Compulsory Input Parameters

1: $\mathbf{x}(lx)$ – REAL (KIND=nag_wp) array

$x \in X$, the set of arguments of the series.

Constraint: $\mathbf{xmin} \leq \mathbf{x}(i) \leq \mathbf{xmax}$, for $i = 1, 2, \dots, lx$.

- 2: **xmin** – REAL (KIND=nag_wp)
 3: **xmax** – REAL (KIND=nag_wp)

The lower and upper end points respectively of the interval $[x_{\min}, x_{\max}]$. The Chebyshev series representation is in terms of the normalized variable \bar{x} , where

$$\bar{x} = \frac{2x - (x_{\max} + x_{\min})}{x_{\max} - x_{\min}}.$$

Constraint: **xmin** < **xmax**.

- 4: **c(n)** – REAL (KIND=nag_wp) array

c(j) must contain the coefficient c_j of the Chebyshev series, for $j = 1, 2, \dots, n$.

- 5: **s** – INTEGER

Determines the series (see Section 3).

s = 1

The series is general.

s = 2

The series is even.

s = 3

The series is odd.

Constraint: **s** = 1, 2 or 3.

5.2 Optional Input Parameters

- 1: **lx** – INTEGER

Default: the dimension of the array **x**.

The number of evaluation points in X .

Constraint: **lx** ≥ 1 .

- 2: **n** – INTEGER

Default: the dimension of the array **c**.

n , the number of terms in the series.

Constraint: **n** ≥ 1 .

5.3 Output Parameters

- 1: **res(lx)** – REAL (KIND=nag_wp) array

The Chebyshev series evaluated at the set of points X .

- 2: **ifail** – INTEGER

ifail = 0 unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

Constraint: **lx** ≥ 1 .

ifail = 2Constraint: $\mathbf{n} \geq 1$.**ifail = 3**Constraint: $\mathbf{s} = 1, 2$ or 3 .**ifail = 4**Constraint: **xmin < xmax**.**ifail = 5**Constraint: **xmin ≤ x(i) ≤ xmax**, for all i .**ifail = -99**

An unexpected error has been triggered by this routine. Please contact NAG.

ifail = -399

Your licence key may have expired or may not have been installed correctly.

ifail = -999

Dynamic memory allocation failed.

7 Accuracy

There may be a loss of significant figures due to cancellation between terms. However, provided that n is not too large, nag_sum_chebyshev (c06dc) yields results which differ little from the best attainable for the available ***machine precision***.

8 Further Comments

The time taken increases with n .

nag_sum_chebyshev (c06dc) has been prepared in the present form to complement a number of integral equation solving functions which use Chebyshev series methods, e.g., nag_inteq_fredholm2_split (d05aa) and nag_inteq_fredholm2_smooth (d05ab).

9 Example

This example evaluates

$$0.5 + T_1(x) + 0.5T_2(x) + 0.25T_3(x)$$

at the points $X = [0.5, 1.0, -0.2]$.

9.1 Program Text

```
function c06dc_example

fprintf('c06dc example results\n\n');

x = [0.5, 1.0, -0.2];
xmin = -1;
xmax = 1;
s = nag_int(1);
c = [1.0, 1.0, 0.5, 0.25];

[res, ifail] = c06dc(x, xmin, xmax, c, s);
fprintf('\nSums: \n');
disp(res);
```

9.2 Program Results

c06dc example results

Sums:

0.5000
2.2500
-0.0180
