NAG Toolbox

nag_sum_convcorr_real (c06fk)

1 Purpose

nag_sum_convcorr_real (c06fk) calculates the circular convolution or correlation of two real vectors of period n (using a work array for extra speed).

2 Syntax

```
[x, y, ifail] = nag_sum_convcorr_real(job, x, y, 'n', n)
[x, y, ifail] = c06fk(job, x, y, 'n', n)
```

3 Description

nag_sum_convcorr_real (c06fk) computes:

if **job** $= 1$, the discrete **convolution** of x and y, defined by

$$z_k = \sum_{j=0}^{n-1} x_j y_{k-j} = \sum_{j=0}^{n-1} x_{k-j} y_j;$$

if **job** $= 2$, the discrete **correlation** of x and y defined by

$$w_k = \sum_{j=0}^{n-1} x_j y_{k+j}.$$

Here x and y are real vectors, assumed to be periodic, with period n, i.e., $x_j = x_{j\pm n} = x_{j\pm 2n} = \ldots$; z and w are then also periodic with period n.

Note: this usage of the terms 'convolution' and 'correlation' is taken from Brigham (1974). The term 'convolution' is sometimes used to denote both these computations.

If $\hat{x}$, $\hat{y}$, $\hat{z}$ and $\hat{w}$ are the discrete Fourier transforms of these sequences, i.e.,

$$\hat{x}_k = \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} x_j \times \exp\left(-i\frac{2\pi jk}{n}\right), \text{ etc.,}$$

then $\hat{z}_k = \sqrt{n}.\hat{x}_k \hat{y}_k$ and $\hat{w}_k = \sqrt{n}.\bar{\hat{x}}_k \hat{y}_k$ (the bar denoting complex conjugate).

This function calls the same auxiliary functions as nag_sum_fft_realherm_1d (c06pa) to compute discrete Fourier transforms.

4 References

Brigham E O (1974) *The Fast Fourier Transform* Prentice–Hall

5 Parameters

5.1 Compulsory Input Parameters

1: **job** – INTEGER

The computation to be performed.

job $= 1$

$$z_k = \sum_{j=0}^{n-1} x_j y_{k-j} \text{ (convolution);}$$

job $= 2$

$$w_k = \sum_{j=0}^{n-1} x_j y_{k+j} \text{ (correlation).}$$

Constraint: **job** $= 1$ or 2.

2: $\mathbf{x}(\mathbf{n})$ – REAL (KIND=nag_wp) array

The elements of one period of the vector x. If **x** is declared with bounds $(0 : \mathbf{n} - 1)$ in the function from which nag_sum_convcorr_real (c06fk) is called, then $\mathbf{x}(j)$ must contain x_j, for $j = 0, 1, \ldots, n - 1$.

3: $\mathbf{y}(\mathbf{n})$ – REAL (KIND=nag_wp) array

The elements of one period of the vector y. If **y** is declared with bounds $(0 : \mathbf{n} - 1)$ in the function from which nag_sum_convcorr_real (c06fk) is called, then $\mathbf{y}(j)$ must contain y_j, for $j = 0, 1, \ldots, n - 1$.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the dimension of the arrays **x**, **y**. (An error is raised if these dimensions are not equal.)

n, the number of values in one period of the vectors **x** and **y**.

Constraint: $\mathbf{n} > 1$.

5.3 Output Parameters

1: $\mathbf{x}(\mathbf{n})$ – REAL (KIND=nag_wp) array

The corresponding elements of the discrete convolution or correlation.

2: $\mathbf{y}(\mathbf{n})$ – REAL (KIND=nag_wp) array

The discrete Fourier transform of the convolution or correlation returned in the array **x**; the transform is stored in Hermitian form; if the components of the transform z_k are written as $a_k + ib_k$, then for $0 \leq k \leq n/2$, a_k is contained in $\mathbf{y}(k)$, and for $1 \leq k \leq n/2 - 1$, b_k is contained in $\mathbf{y}(n - k)$. (See also Section 2.1.2 in the C06 Chapter Introduction.)

3: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 3$

On entry, $\mathbf{n} \leq 1$.

ifail $= 4$

On entry, $\mathbf{job} \neq 1$ or 2.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The results should be accurate to within a small multiple of the ***machine precision***.

8 Further Comments

The time taken is approximately proportional to $n \times \log(n)$, but also depends on the factorization of n. nag_sum_convcorr_real (c06fk) is faster if the only prime factors of n are 2, 3 or 5; and fastest of all if n is a power of 2.

9 Example

This example reads in the elements of one period of two real vectors x and y, and prints their discrete convolution and correlation (as computed by nag_sum_convcorr_real (c06fk)). In realistic computations the number of data values would be much larger.

9.1 Program Text

```
function c06fk_example

fprintf('c06fk example results\n\n');

x = [1;    1;    1;    1;    1;    0;    0;    0;    0];
y = [0.5;  0.5;  0.5;  0.5;  0;    0;    0;    0;    0];

job = nag_int(1);
[conv, tconv, ifail] = c06fk(job, x, y);

job = nag_int(2);
[corr, tcorr, ifail] = c06fk(job, x, y);

result = [conv corr];
disp('Convolution  Correlation');
disp(result);
```

9.2 Program Results

```
c06fk example results

Convolution  Correlation
    0.5000    2.0000
    1.0000    1.5000
    1.5000    1.0000
    2.0000    0.5000
    2.0000    0.0000
    1.5000    0.5000
    1.0000    1.0000
    0.5000    1.5000
    0.0000    2.0000
```