NAG Toolbox

nag_sum_invlaplace_weeks_eval (c06lc)

1 Purpose

nag_sum_invlaplace_weeks_eval (c06lc) evaluates an inverse Laplace transform at a given point, using the expansion coefficients computed by nag_sum_invlaplace_weeks (c06lb).

2 Syntax

```
[finv, ifail] = nag_sum_invlaplace_weeks_eval(t, sigma, b, acoef, errvec, 'm', m)
[finv, ifail] = c06lc(t, sigma, b, acoef, errvec, 'm', m)
```

3 Description

nag_sum_invlaplace_weeks_eval (c06lc) is designed to be used following a call to nag_sum_invlaplace_weeks (c06lb), which computes an inverse Laplace transform by representing it as a Laguerre expansion of the form:

$$\tilde{f}(t) = e^{\sigma t}\sum_{i=0}^{m-1} a_i e^{-bt/2} L_i(bt), \quad \sigma > \sigma_O, \quad b > 0$$

where $L_i(x)$ is the Laguerre polynomial of degree i.

This function simply evaluates the above expansion for a specified value of t.

nag_sum_invlaplace_weeks_eval (c06lc) is derived from the function MODUL2 in Garbow *et al.* (1988)

4 References

Garbow B S, Giunta G, Lyness J N and Murli A (1988) Algorithm 662: A Fortran software package for the numerical inversion of the Laplace transform based on Weeks' method *ACM Trans. Math. Software* **14** 171–176

5 Parameters

5.1 Compulsory Input Parameters

1: **t** – REAL (KIND=nag_wp)

The value t for which the inverse Laplace transform $f(t)$ must be evaluated.

2: **sigma** – REAL (KIND=nag_wp)

3: **b** – REAL (KIND=nag_wp)

4: **acoef**(**m**) – REAL (KIND=nag_wp) array

5: **errvec**(**8**) – REAL (KIND=nag_wp) array

sigma, **b**, **m**, **acoef** and **errvec** must be unchanged from the previous call of nag_sum_invlaplace_weeks (c06lb).

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the dimension of the array **acoef**.

sigma, **b**, **m**, **acoef** and **errvec** must be unchanged from the previous call of nag_sum_invlaplace_weeks (c06lb).

5.3 Output Parameters

1: **finv** – REAL (KIND=nag_wp)

The approximation to the inverse Laplace transform at t.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$ (*warning*)

The approximation to $f(t)$ is too large to be representable: **finv** is set to 0.0.

ifail $= 2$ (*warning*)

The approximation to $f(t)$ is too small to be representable: **finv** is set to 0.0.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The error estimate returned by nag_sum_invlaplace_weeks (c06lb) in **errvec**(1) has been found in practice to be a highly reliable bound on the pseudo-error $\left|f(t) - \tilde{f}(t)\right|e^{-\sigma t}$.

8 Further Comments

nag_sum_invlaplace_weeks_eval (c06lc) is primarily designed to evaluate $\tilde{f}(t)$ when $t > 0$. When $t \leq 0$, the result approximates the analytic continuation of $f(t)$; the approximation becomes progressively poorer as t becomes more negative.

9 Example

See example for nag_sum_invlaplace_weeks (c06lb).

9.1 Program Text

```
     function c06lc_example

fprintf('c06lc example results\n\n');

sigma0 = 3;
sigma = 0;
b = 0;
epstol = 1e-05;
[sigmaOut, bOut, m, acoef, errvec, ifail] = ...
    c06lb(@f, sigma0, sigma, b, epstol, 'mmax', nag_int(512));

fprintf('\nNo. of coefficients returned by c06lb = %d\n\n', m);
fprintf('                   Computed           Exact       Pseudo\n');
fprintf('        t              f(t)            f(t)        error\n');
for j = 0:5
  [finv, ifail] = c06lc(j, sigmaOut, bOut, acoef, errvec);
  exact = sinh(3*j);
  pserr = abs(finv-exact)/exp(sigmaOut*j);
  fprintf(' %10.2f %15.4f %15.4f %12.1g\n',j, finv, exact, pserr);
end

function [f] = f(s)
% Evaluate the Laplace transform function.
f=3.0/(s^2-9.0);
if isreal(f)
    f=complex(f);
end
```

9.2 Program Results

```
     c06lc example results

No. of coefficients returned by c06lb = 64

                  Computed           Exact       Pseudo
        t             f(t)            f(t)        error
     0.00           0.0000          0.0000        2e-09
     1.00          10.0179         10.0179        2e-09
     2.00         201.7132        201.7132        1e-10
     3.00        4051.5420       4051.5419        1e-09
     4.00       81377.3949      81377.3957        3e-10
     5.00     1634508.5023    1634508.6862        2e-09
```