NAG Toolbox

nag_pde_2d_triangulate (d03ma)

1 Purpose

nag_pde_2d_triangulate (d03ma) places a triangular mesh over a given two-dimensional region. The region may have any shape, including one with holes.

2 Syntax

```
[npts, places, indx, ifail] = nag_pde_2d_triangulate(h, m, n, nb, sdindx, isin)
[npts, places, indx, ifail] = d03ma(h, m, n, nb, sdindx, isin)
```

3 Description

nag_pde_2d_triangulate (d03ma) begins with a uniform triangular grid as shown in Figure 1 and assumes that the region to be triangulated lies within the rectangle given by the inequalities

$$0 < x < \sqrt{3}(m-1)h, \quad 0 < y < (n-1)h.$$

This rectangle is drawn in bold in Figure 1. The region is specified by the **isin** which must determine whether any given point (x, y) lies in the region. The uniform grid is processed column-wise, with (x_1, y_1) preceding (x_2, y_2) if $x_1 < x_2$ or $x_1 = x_2$, $y_1 < y_2$. Points near the boundary are moved onto it and points well outside the boundary are omitted. The direction of movement is chosen to avoid pathologically thin triangles. The points accepted are numbered in exactly the same order as the corresponding points of the uniform grid were scanned. The output consists of the x, y coordinates of all grid points and integers indicating whether they are internal and to which other points they are joined by triangle sides.

The mesh size h must be chosen small enough for the essential features of the region to be apparent from testing all points of the original uniform grid for being inside the region. For instance if any hole is within $2h$ of another hole or the outer boundary then a triangle may be found with all vertices within $\frac{1}{2}h$ of a boundary. Such a triangle is taken to be external to the region so the effect will be to join the hole to another hole or to the external region.

Further details of the algorithm are given in the references.

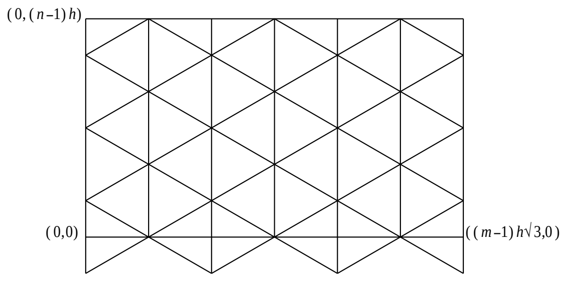


Figure 1

4 References

Reid J K (1970) Fortran subroutines for the solutions of Laplace's equation over a general routine in two dimensions *Harwell Report TP422*

Reid J K (1972) On the construction and convergence of a finite-element solution of Laplace's equation *J. Instr. Math. Appl.* **9** 1–13

5 Parameters

5.1 Compulsory Input Parameters

1: **h** – REAL (KIND=nag_wp)

h, the required length for the sides of the triangles of the uniform mesh.

2: **m** – INTEGER

3: **n** – INTEGER

Values m and n such that all points (x, y) inside the region satisfy the inequalities

$$0 \le x \le \sqrt{3}(m-1)h,$$
$$0 \le y \le (n-1)h.$$

Constraint: $\mathbf{m} = \mathbf{n} > 2$.

4: **nb** – INTEGER

The number of times a triangle side is bisected to find a point on the boundary. A value of 10 is adequate for most purposes (see Section 7).

Constraint: $\mathbf{nb} \ge 1$.

5: **sdindx** – INTEGER

An upper bound on the number of points in the triangulation. The actual value will be returned in **npts**.

6: **isin** – INTEGER FUNCTION, supplied by the user.

isin must return the value 1 if the given point (**x**,**y**) lies inside the region, and 0 if it lies outside.

```
[result] = isin(x, y)
```

Input Parameters

1: **x** – REAL (KIND=nag_wp)

2: **y** – REAL (KIND=nag_wp)

The coordinates of the given point.

Output Parameters

1: **result**

The value 1 if the given point (**x**,**y**) lies inside the region, and 0 if it lies outside.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **npts** – INTEGER

The number of points in the triangulation.

2: **places**$(\mathbf{2}, \mathbf{sdindx})$ – REAL (KIND=nag_wp) array

The x and y coordinates respectively of the ith point of the triangulation.

3: **indx**$(\mathbf{4}, \textbf{sdindx})$ – INTEGER array

indx$(1, i)$ contains i if point i is inside the region and $-i$ if it is on the boundary. For each triangle side between points i and j with $j > i$, **indx**(k, i), $k > 1$, contains j or $-j$ according to whether point j is internal or on the boundary. There can never be more than three such points. If there are less, then some values **indx**(k, i), $k > 1$, are zero.

4: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

sdindx is too small.

ifail $= 2$

A point inside the region violates one of the constraints (see arguments **m** and **n**).

ifail $= 3$

$sddist$ is too small.

ifail $= 4$

m ≤ 2.

ifail $= 5$

n ≤ 2.

ifail $= 6$

nb ≤ 0.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

Points are moved onto the boundary by bisecting a triangle side **nb** times. The accuracy is therefore $h \times 2^{-\textbf{nb}}$.

8 Further Comments

The time taken is approximately proportional to $m \times n$.

9 Example

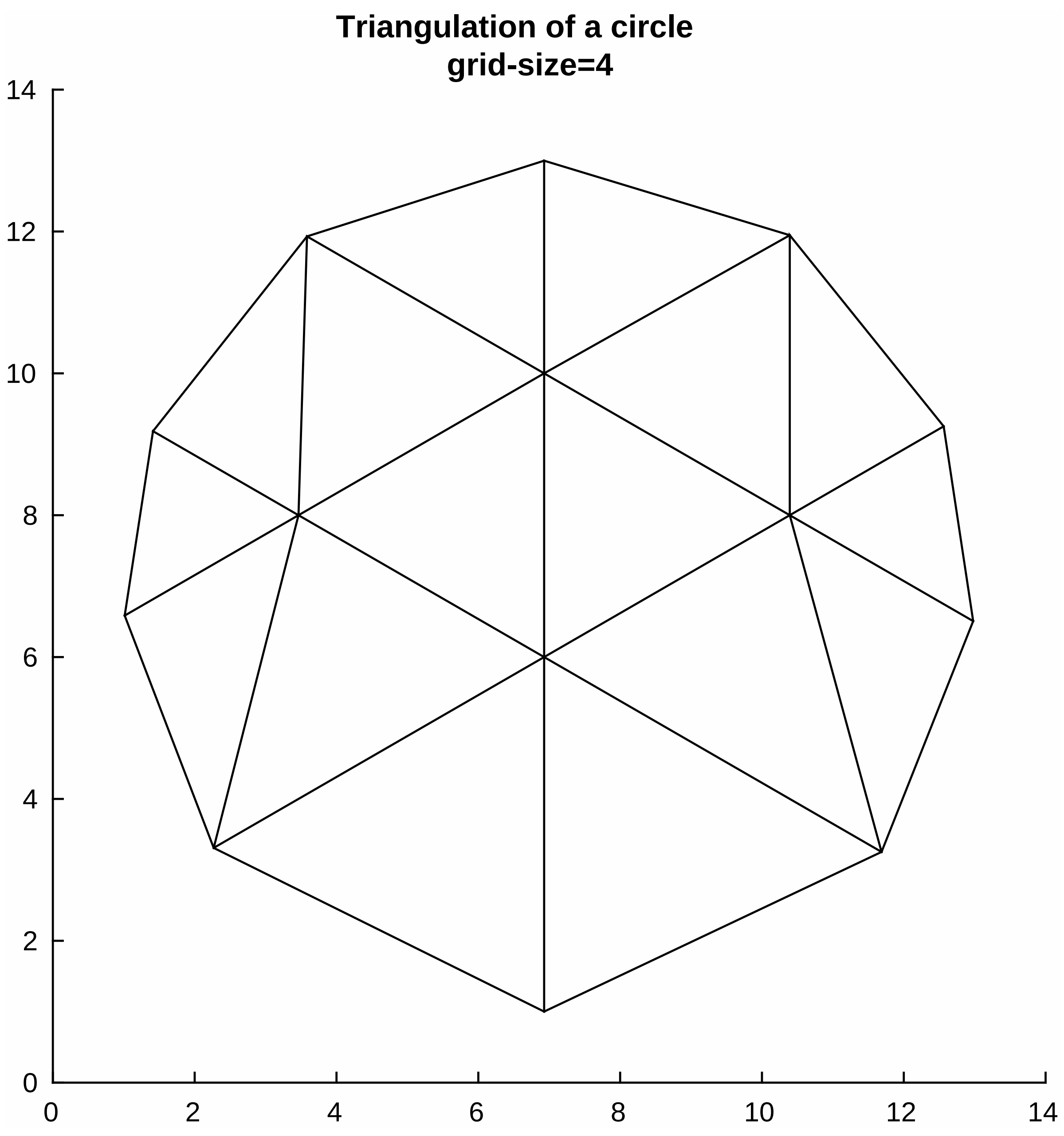
The following program triangulates the circle with centre $(7.0, 7.0)$ and radius 6.0 using a basic grid size $h = 4.0$.

9.1 Program Text

```
     function d03ma_example

fprintf('d03ma example results\n\n');

h = 4;
m = nag_int(3);
n = nag_int(5);
nb = nag_int(10);
sdindx = nag_int(100);
[npts, places, index, ifail] = d03ma( ...
                                      h, m, n, nb, sdindx, @isin);

fprintf('Number of points = %4d\n',npts);

fig1 = figure;
k = 0;
hold on
for i = 1:npts
  for j=2:4
    % lines connecting from node i
     if(index(j,i)~=0)
       k = k + 1;
       l = abs(index(j,i));
       x(1) = places(1,i);
       x(2) = places(1,l);
       y(1) = places(2,i);
       y(2) = places(2,l);
       plot(x,y)
     end
  end
end
axis square;
title({'Triangulation of a circle','grid-size=4'});
hold off;

fprintf('Number of edges  = %4d\n',k);

function result = isin(x,y)
  if ((x-7)^2+(y-7)^2 <= 36)
    result = nag_int(1);
  else
    result = nag_int(0);
  end
```

9.2 Program Results

```
     d03ma example results

Number of points =   14
Number of edges  =   29
```


Triangulation of a circle
grid-size=4
0
2
4
6
8
10
12
14