

NAG Toolbox

nag_pde_1d_blacksholes_means (d03ne)

1 Purpose

nag_pde_1d_blacksholes_means (d03ne) computes average values of a continuous function of time over the remaining life of an option. It is used together with nag_pde_1d_blacksholes_closed (d03nd) to value options with time-dependent arguments.

2 Syntax

```
[phiav, ifail] = nag_pde_1d_blacksholes_means(t0, tmat, td, phid, 'ntd', ntd)
[phiav, ifail] = d03ne(t0, tmat, td, phid, 'ntd', ntd)
```

3 Description

nag_pde_1d_blacksholes_means (d03ne) computes the quantities

$$\phi(t_0), \quad \hat{\phi} = \frac{1}{T-t_0} \int_{t_0}^T \phi(\zeta) d\zeta, \quad \bar{\phi} = \left(\frac{1}{T-t_0} \int_{t_0}^T \phi^2(\zeta) d\zeta \right)^{1/2}$$

from a given set of values **phid** of a continuous time-dependent function $\phi(t)$ at a set of discrete points **td**, where t_0 is the current time and T is the maturity time. Thus $\hat{\phi}$ and $\bar{\phi}$ are first and second order averages of ϕ over the remaining life of an option.

The function may be used in conjunction with nag_pde_1d_blacksholes_closed (d03nd) in order to value an option in the case where the risk-free interest rate r , the continuous dividend q , or the stock volatility σ is time-dependent and is described by values at a set of discrete times (see Section 9.2). This is illustrated in Section 10.

4 References

None.

5 Parameters

5.1 Compulsory Input Parameters

1: **t0** – REAL (KIND=nag_wp)

The current time t_0 .

Constraint: $\text{td}(1) \leq \text{t0} \leq \text{td(ntd)}$.

2: **tmat** – REAL (KIND=nag_wp)

The maturity time T .

Constraint: $\text{td}(1) \leq \text{tmat} \leq \text{td(ntd)}$.

3: **td(ntd)** – REAL (KIND=nag_wp) array

The discrete times at which ϕ is specified.

Constraint: $\text{td}(1) < \text{td}(2) < \dots < \text{td(ntd)}$.

4: **phid(ntd)** – REAL (KIND=nag_wp) array
phid(*i*) must contain the value of ϕ at time **td**(*i*), for $i = 1, 2, \dots, ntd$.

5.2 Optional Input Parameters

1: **ntd** – INTEGER

Default: the dimension of the arrays **td**, **phid**. (An error is raised if these dimensions are not equal.)

The number of discrete times at which ϕ is given.

Constraint: **ntd** ≥ 2 .

5.3 Output Parameters

1: **phiav(3)** – REAL (KIND=nag_wp) array

phiav(1) contains the value of ϕ interpolated to t_0 , **phiav**(2) contains the first-order average $\hat{\phi}$ and **phiav**(3) contains the second-order average $\bar{\phi}$, where:

$$\hat{\phi} = \frac{1}{T-t_0} \int_{t_0}^T \phi(\zeta) d\zeta, \quad \bar{\phi} = \left(\frac{1}{T-t_0} \int_{t_0}^T \phi^2(\zeta) d\zeta \right)^{1/2}.$$

2: **ifail** – INTEGER

ifail = 0 unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, **t0** lies outside the range [**td**(1), **td**(**ntd**)],
or **tmat** lies outside the range [**td**(1), **td**(**ntd**)],
or **ntd** < 2 ,
or **td** badly ordered,
or *lwork* $< 9 \times \text{ntd} + 24$.

ifail = 2

Unexpected failure in internal call to `nag_interp_1d_spline` (e01ba) or `nag_fit_1dspline_eval` (e02bb).

ifail = -99

An unexpected error has been triggered by this routine. Please contact NAG.

ifail = -399

Your licence key may have expired or may not have been installed correctly.

ifail = -999

Dynamic memory allocation failed.

7 Accuracy

If $\phi \in C^4[t_0, T]$ then the error in the approximation of $\phi(t_0)$ and $\hat{\phi}$ is $O(H^4)$, where $H = \max_i(T(i+1) - T(i))$, for $i = 1, 2, \dots, ntd - 1$. The approximation is exact for polynomials of degree up to 3.

The third quantity $\bar{\phi}$ is $O(H^2)$, and exact for linear functions.

8 Further Comments

8.1 Timing

The time taken is proportional to **ntd**.

8.2 Use with nag_pde_1d_blacksholes_closed (d03nd)

Suppose you wish to evaluate the analytic solution of the Black–Scholes equation in the case when the risk-free interest rate r is a known function of time, and is represented as a set of values at discrete times. A call to nag_pde_1d_blacksholes_means (d03ne) providing these values in **phid** produces an output array **phiav** suitable for use as the argument **r** in a subsequent call to nag_pde_1d_blacksholes_closed (d03nd).

Time-dependent values of the continuous dividend Q and the volatility σ may be handled in the same way.

8.3 Algorithmic Details

The **ntd** data points are fitted with a cubic B-spline using the function nag_interp_1d_spline (e01ba). Evaluation is then performed using nag_fit_1dspline_eval (e02bb), and the definite integrals are computed using direct integration of the cubic splines in each interval. The special case of $T = t_o$ is handled by interpolating ϕ at that point.

9 Example

This example demonstrates the use of the function in conjunction with nag_pde_1d_blacksholes_closed (d03nd) to solve the Black–Scholes equation for valuation of a 5-month American call option on a non-dividend-paying stock with an exercise price of \$50. The risk-free interest rate varies linearly with time and the stock volatility has a quadratic variation. Since these functions are integrated exactly by nag_pde_1d_blacksholes_means (d03ne) the solution of the Black–Scholes equation by nag_pde_1d_blacksholes_closed (d03nd) is also exact.

The option is valued at a range of times and stock prices.

9.1 Program Text

```
function d03ne_example

fprintf('d03ne example results\n\n');

tmat = 0.4166667;
td = [0:0.1:0.5];
rd = [0.1:0.01:0.15];
sigd = [0.3 0.46 0.54 0.54 0.36 0.3];

% American 5-month call option, exercise price 50

kopt = nag_int(2);
x = 50;
ns = 21; nt = 4;
s_beg = 0; t_beg = 0;
s_end = 100; t_end = 0.125;
tmat = 0.4166667;
tdpar = [true; false; true];
q = [0];

% Discretize s and t
ds = (s_end-s_beg)/(ns-1);
dt = (t_end-t_beg)/(nt-1);
s = [s_beg:ds:s_end];
t = [t_beg:dt:t_end];
```

```

f = zeros(ns,nt);
theta = f; delta = f; gamma = f; lambda = f; rho = f;

% Loop over times and prices
for j = 1:nt

    % Find average values of r and sigma
    [ra, ifail] = d03ne( ...
        t(j), tmat, td, rd);
    [siga, ifail] = d03ne( ...
        t(j), tmat, td, sigd);

    for i = 1:ns
        [f(i,j),theta(i,j),delta(i,j),gamma(i,j),lambda(i,j),rho(i,j),ifail] = ...
            d03nd( ...
                kopt, x, s(i), t(j), tmat, tdpar, ra, q, siga);
    end
end

% Tabulate option values only
print_greek(ns,nt,tmat,s,t,'Option Values',f);
% print_greek(ns,nt,tmat,s,t,'Theta',theta);
% print_greek(ns,nt,tmat,s,t,'Delta',delta);
% print_greek(ns,nt,tmat,s,t,'Gamma',gamma);
% print_greek(ns,nt,tmat,s,t,'Lambda',lambda);
% print_greek(ns,nt,tmat,s,t,'Rho',rho);

% plot initial and final option values and greeks
fig1 = figure;
plot(s,f(:,1),s,theta(:,1),s,delta(:,1),s,gamma(:,1),s,lambda(:,1),s,rho(:,1));
legend('value','theta','delta','gamma','lambda','rho','Location','NorthWest');
title('Option values and greeks at 5 months to maturity');
xlabel('stock price');
ylabel('values and derivatives');
fig2 = figure;
plot(s,f(:,4),s,theta(:,4),s,delta(:,4),s,gamma(:,4),s,lambda(:,4),s,rho(:,4));
legend('value','theta','delta','gamma','lambda','rho','Location','NorthWest');
title('Option values and greeks at 3.5 months to maturity');
xlabel('stock price');
ylabel('values and derivatives');

function print_greek(ns,nt,tmat,s,t,grname,greek)

    fprintf('\n%s\n\n',grname);
    fprintf(' Stock Price | Time to Maturity (months)\n');
    fprintf('%16s %12.4e%12.4e%12.4e\n', '|', 12*(tmat-t));
    fprintf('%15s+%48s\n', '-----', ...
        '-----');
    for i = 1:ns
        fprintf('%12.4e%4s %12.4e%12.4e%12.4e\n', s(i), '|', greek(i,:));
    end

```

9.2 Program Results

d03ne example results

Option Values

Stock Price		Time to Maturity (months)		
		5.0000e+00	4.5000e+00	4.0000e+00
0.0000e+00		0.0000e+00	0.0000e+00	0.0000e+00
5.0000e+00		8.3570e-14	1.6081e-14	1.1216e-15
1.0000e+01		2.5775e-07	1.0991e-07	2.8081e-08
1.5000e+01		1.7581e-04	1.0511e-04	4.6513e-05
2.0000e+01		6.9193e-03	4.9696e-03	2.9591e-03
2.5000e+01		7.0752e-02	5.6767e-02	4.0397e-02
3.0000e+01		3.4255e-01	2.9499e-01	2.3506e-01
3.5000e+01		1.0512e+00	9.4849e-01	8.1382e-01

4.0000e+01		2.3997e+00	2.2341e+00	2.0134e+00	1.7424e+00
4.5000e+01		4.4829e+00	4.2630e+00	3.9702e+00	3.6055e+00
5.0000e+01		7.2786e+00	7.0226e+00	6.6859e+00	6.2677e+00
5.5000e+01		1.0687e+01	1.0414e+01	1.0063e+01	9.6324e+00
6.0000e+01		1.4580e+01	1.4305e+01	1.3959e+01	1.3546e+01
6.5000e+01		1.8832e+01	1.8563e+01	1.8236e+01	1.7855e+01
7.0000e+01		2.3337e+01	2.3079e+01	2.2774e+01	2.2429e+01
7.5000e+01		2.8016e+01	2.7768e+01	2.7485e+01	2.7173e+01
8.0000e+01		3.2809e+01	3.2573e+01	3.2308e+01	3.2022e+01
8.5000e+01		3.7678e+01	3.7450e+01	3.7201e+01	3.6935e+01
9.0000e+01		4.2595e+01	4.2374e+01	4.2136e+01	4.1885e+01
9.5000e+01		4.7543e+01	4.7327e+01	4.7097e+01	4.6856e+01
1.0000e+02		5.2510e+01	5.2298e+01	5.2074e+01	5.1840e+01



