NAG Toolbox

nag_pde_1d_parab_euler_hll (d03pw)

1 Purpose

nag_pde_1d_parab_euler_hll (d03pw) calculates a numerical flux function using a modified HLL (Harten–Lax–van Leer) Approximate Riemann Solver for the Euler equations in conservative form. It is designed primarily for use with the upwind discretization schemes nag_pde_1d_parab_convdiff (d03pf), nag_pde_1d_parab_convdiff_dae (d03pl) or nag_pde_1d_parab_convdiff_remesh (d03ps), but may also be applicable to other conservative upwind schemes requiring numerical flux functions.

2 Syntax

```
[flux, ifail] = nag_pde_1d_parab_euler_hll(uleft, uright, gamma)
[flux, ifail] = d03pw(uleft, uright, gamma)
```

3 Description

nag_pde_1d_parab_euler_hll (d03pw) calculates a numerical flux function at a single spatial point using a modified HLL (Harten–Lax–van Leer) Approximate Riemann Solver (see Toro (1992), Toro (1996) and Toro *et al.* (1994)) for the Euler equations (for a perfect gas) in conservative form. You must supply the *left* and *right* solution values at the point where the numerical flux is required, i.e., the initial left and right states of the Riemann problem defined below. In nag_pde_1d_parab_convdiff (d03pf), nag_pde_1d_parab_convdiff_dae (d03pl) and nag_pde_1d_parab_convdiff_remesh (d03ps), the left and right solution values are derived automatically from the solution values at adjacent spatial points and supplied to the function argument **numflx** from which you may call nag_pde_1d_parab_euler_hll (d03pw).

The Euler equations for a perfect gas in conservative form are:

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = 0, \tag{1}$$

with

$$U = \begin{bmatrix} \rho \\ m \\ e \end{bmatrix} \quad \text{and} \quad F = \begin{bmatrix} m \\ \frac{m^2}{\rho} + (\gamma - 1)\left(e - \frac{m^2}{2\rho}\right) \\ \frac{me}{\rho} + \frac{m}{\rho}(\gamma - 1)\left(e - \frac{m^2}{2\rho}\right) \end{bmatrix}, \tag{2}$$

where ρ is the density, m is the momentum, e is the specific total energy and γ is the (constant) ratio of specific heats. The pressure p is given by

$$p = (\gamma - 1)\left(e - \frac{\rho u^2}{2}\right), \tag{3}$$

where $u = m/\rho$ is the velocity.

The function calculates an approximation to the numerical flux function $F(U_L, U_R) = F(U^*(U_L, U_R))$, where $U = U_L$ and $U = U_R$ are the left and right solution values, and $U^*(U_L, U_R)$ is the intermediate state $\omega(0)$ arising from the similarity solution $U(y, t) = \omega(y/t)$ of the Riemann problem defined by

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial y} = 0, \tag{4}$$

with U and F as in (2), and initial piecewise constant values $U = U_L$ for $y < 0$ and $U = U_R$ for $y > 0$. The spatial domain is $-\infty < y < \infty$, where $y = 0$ is the point at which the numerical flux is required.

4 References

Toro E F (1992) The weighted average flux method applied to the Euler equations *Phil. Trans. R. Soc. Lond.* **A341** 499–530

Toro E F (1996) *Riemann Solvers and Upwind Methods for Fluid Dynamics* Springer–Verlag

Toro E F, Spruce M and Spears W (1994) Restoration of the contact surface in the HLL Riemann solver *J. Shock Waves* **4** 25–34

5 Parameters

5.1 Compulsory Input Parameters

1: **uleft**(**3**) – REAL (KIND=nag_wp) array

uleft(i) must contain the left value of the component U_i, for $i = 1, 2, 3$. That is, **uleft**(1) must contain the left value of ρ, **uleft**(2) must contain the left value of m and **uleft**(3) must contain the left value of e.

Constraints:

uleft(1) ≥ 0.0;
Left pressure, $pl \geq 0.0$, where pl is calculated using (3).

2: **uright**(**3**) – REAL (KIND=nag_wp) array

uright(i) must contain the right value of the component U_i, for $i = 1, 2, 3$. That is, **uright**(1) must contain the right value of ρ, **uright**(2) must contain the right value of m and **uright**(3) must contain the right value of e.

Constraints:

uright(1) ≥ 0.0;
Right pressure, $pr \geq 0.0$, where pr is calculated using (3).

3: **gamma** – REAL (KIND=nag_wp)

The ratio of specific heats, γ.

Constraint: **gamma** > 0.0.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **flux**(**3**) – REAL (KIND=nag_wp) array

flux(i) contains the numerical flux component $\hat{F}_i$, for $i = 1, 2, 3$.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **gamma** ≤ 0.0.

ifail $= 2$

On entry, the left and/or right density or derived pressure value is less than 0.0.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

nag_pde_1d_parab_euler_hll (d03pw) performs an exact calculation of the HLL (Harten–Lax–van Leer) numerical flux function, and so the result will be accurate to ***machine precision***.

8 Further Comments

nag_pde_1d_parab_euler_hll (d03pw) must only be used to calculate the numerical flux for the Euler equations in exactly the form given by (2), with **uleft**(i) and **uright**(i) containing the left and right values of ρ, m and e, for $i = 1, 2, 3$, respectively. The time taken is independent of the input arguments.

9 Example

This example uses nag_pde_1d_parab_convdiff_dae (d03pl) and nag_pde_1d_parab_euler_hll (d03pw) to solve the Euler equations in the domain $0 \leq x \leq 1$ for $0 < t \leq 0.035$ with initial conditions for the primitive variables $\rho(x,t)$, $u(x,t)$ and $p(x,t)$ given by

$$\begin{array}{llll} \rho(x,0) = 5.99924, & u(x,0) = 19.5975, & p(x,0) = 460.894, & \text{for } x < 0.5, \\ \rho(x,0) = 5.99242, & u(x,0) = -6.19633, & p(x,0) = 46.095, & \text{for } x > 0.5. \end{array}$$

This test problem is taken from Toro (1996) and its solution represents the collision of two strong shocks travelling in opposite directions, consisting of a left facing shock (travelling slowly to the right), a right travelling contact discontinuity and a right travelling shock wave. There is an exact solution to this problem (see Toro (1996)) but the calculation is lengthy and has therefore been omitted.

9.1 Program Text

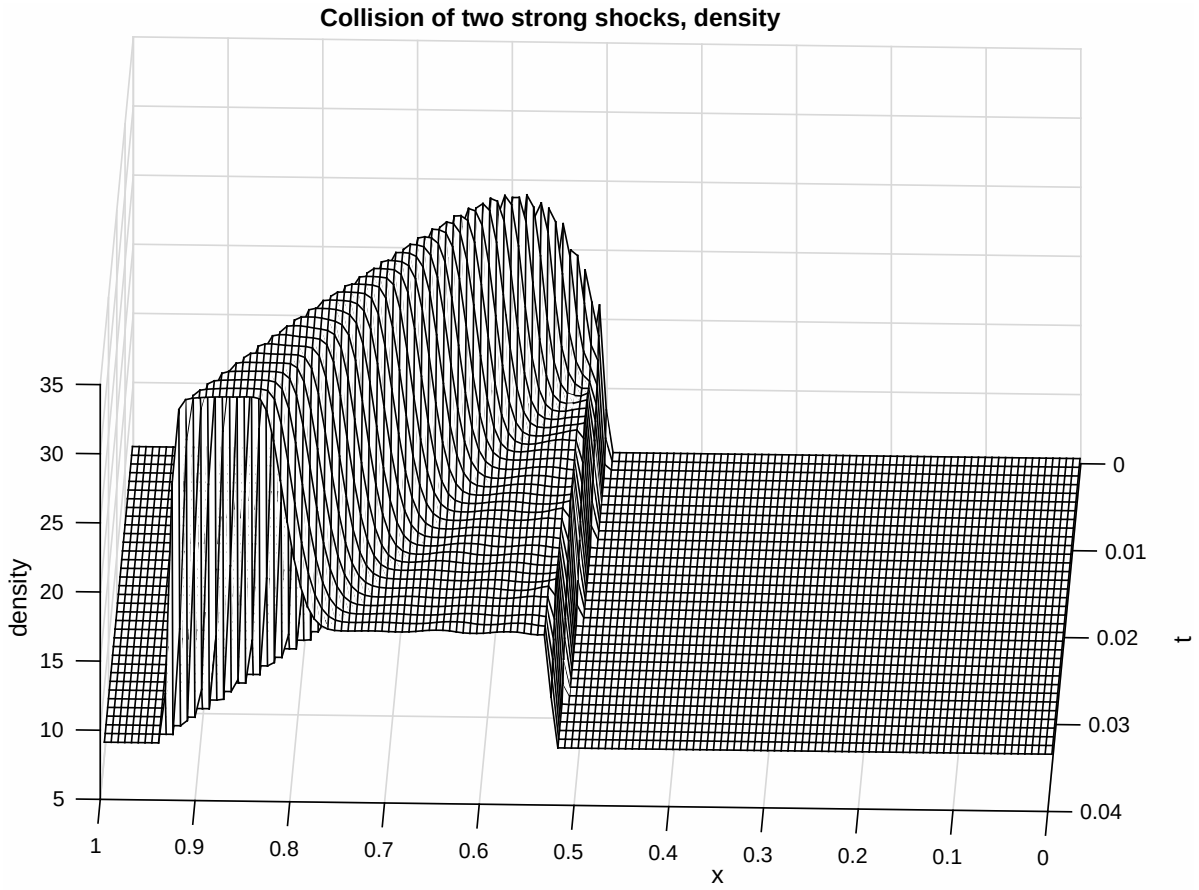
```
     function d03pw_example

fprintf('d03pw example results\n\n');

global gamma rl0 rr0 ul0 ur0 el0 er0;

% Problem parameters
alpha_l = 460.894;
alpha_r = 46.095;
beta_l  = 19.5975;
beta_r  =  6.19633;
gamma  =   1.4;
rl0    =   5.99924;
rr0    =   5.99242;
ul0    = 117.5701059;
ur0    = -37.1310118186;
el0    = alpha_l/(gamma-1) + rl0*beta_l^2/2;
er0    = alpha_r/(gamma-1) + rr0*beta_r^2/2;

npde  = nag_int(3);
npts  = nag_int(141);
```

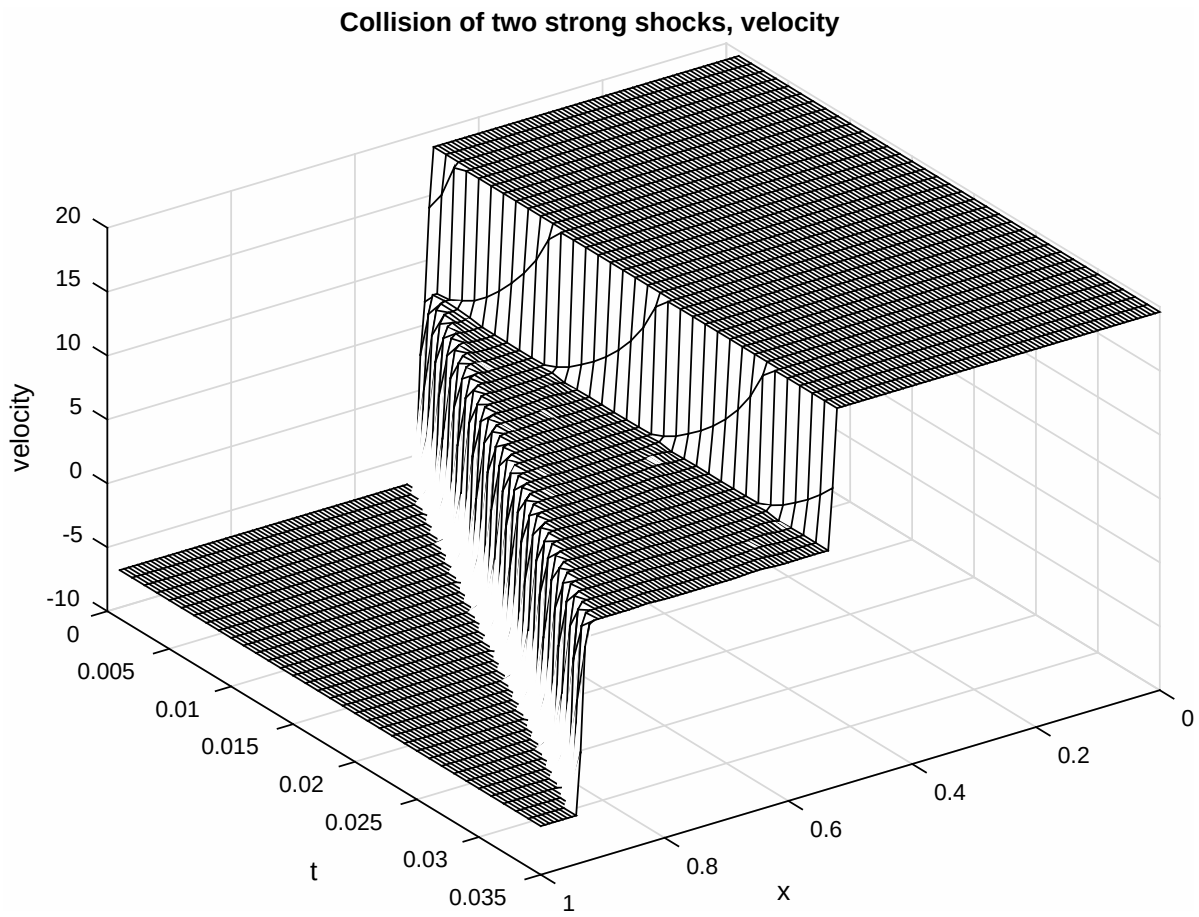
```
ncode = nag_int(0);
nxi   = nag_int(0);
neqn  = npde*npts+ncode;
ts    = 0;
xi    = [];
itol = nag_int(1);
atol = [0.005];
rtol = [0.0005];
norm_p = '2';
laopt = 'B';
algopt = zeros(30,1);
algopt(1) = 2;
algopt(6) = 2;
algopt(7) = 2;
algopt(13) = 0.005;
rsave  = zeros(21000, 1);
isave  = zeros(25700, 1, nag_int_name);
itask  = nag_int(1);
itrace = nag_int(0);
ind    = nag_int(0);

% Initial mesh and solution
dx = 1/(double(npts)-1);
x  = [0:dx:1];
u = uvinit(x);

u1sol = zeros(35,npts);
u2sol = zeros(35,npts);
u3sol = zeros(35,npts);
xsol = zeros(35,npts);
tsol = zeros(35,npts);

for j=1:35
  tout = 0.001*j;
  [ts, u, rsave, isave, ind, ifail] = ...
  d03pl( ...
         npde, ts, tout, 'd03plp', @numflx, @bndary, u, x, ncode, ...
         'd03pek', xi, rtol, atol, itol, norm_p, laopt, ...
         algopt, rsave, isave, itask, itrace, ind,'nxi',nxi);

  xsol(j,:) = x;
  tsol(j,:) = ts;
  u1sol(j,:) = u(1,:);
  u2sol(j,:) = u(2,:)./u(1,:);
  u3sol(j,:) = 0.4*u(1,:).*(u(3,:)./u(1,:)-u2sol(j,:).^2/2);

end

nsteps = 50*((isave(1)+25)/50);
nfuncs = 50*((isave(2)+25)/50);
njacs = isave(3);
niters = isave(5);
fprintf('\n Number of time steps           (nearest 50) = %6d\n',nsteps);
fprintf(' Number of function evaluations (nearest 50) = %6d\n',nfuncs);
fprintf(' Number of Jacobian evaluations (nearest  1) = %6d\n',njacs);
fprintf(' Number of iterations           (nearest  1) = %6d\n',niters);

fig1=figure;
mesh(xsol,tsol,u1sol);
title('Collision of two strong shocks, density');
xlabel('x');
ylabel('t');
zlabel('density');
view(182,40);

fig2=figure;
mesh(xsol,tsol,u2sol);
title('Collision of two strong shocks, velocity');
xlabel('x');
ylabel('t');
zlabel('velocity');
```

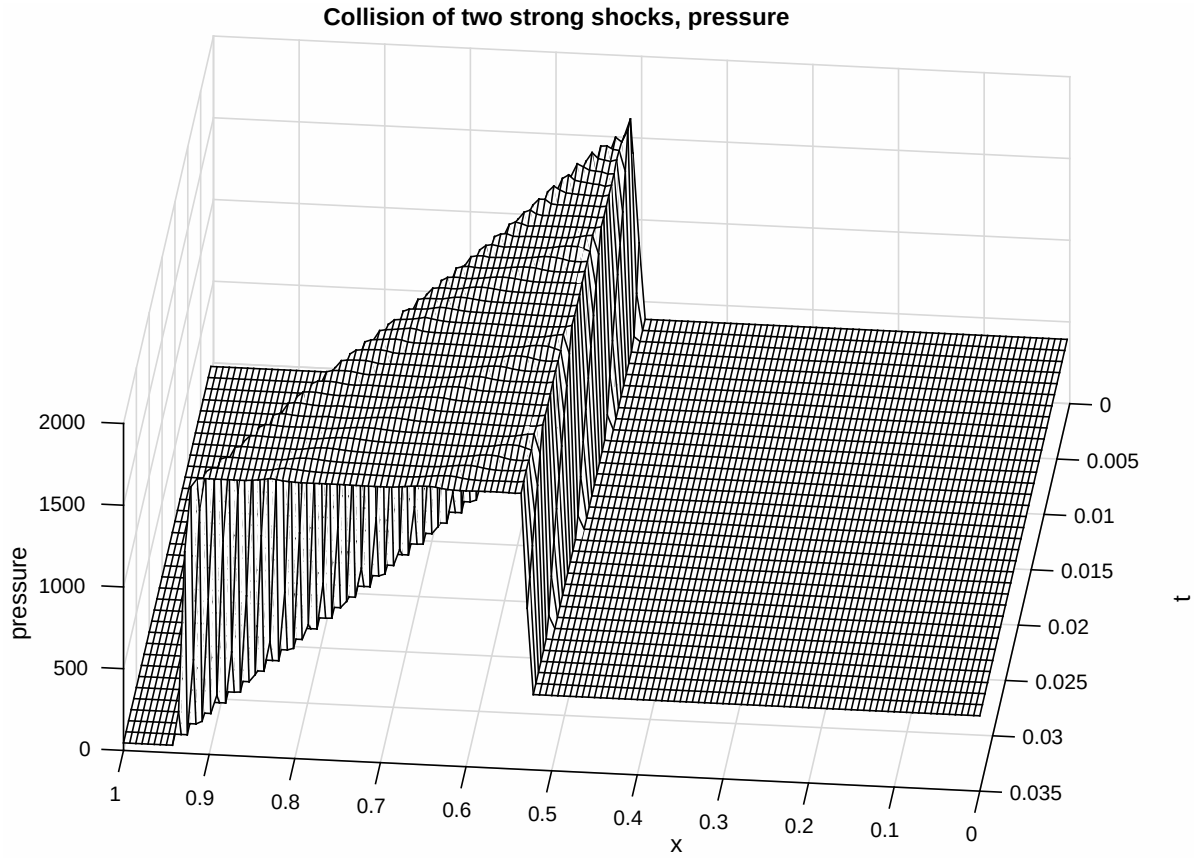
```
view(145,40);

fig3=figure;
mesh(xsol,tsol,u3sol);
title('Collision of two strong shocks, pressure');
xlabel('x');
ylabel('t');
zlabel('pressure');
view(-174,50);

function [g, iresout] = bndary(npde, npts, t, x, u, ncode, ...
                               v, vdot, ibnd, ires)

  global rl0 rr0 ul0 ur0 el0 er0;

  if (ibnd == 0)
    g(1) = u(1,1) - rl0;
    g(2) = u(2,1) - ul0;
    g(3) = u(3,1) - el0;
  else
    g(1) = u(1,npts) - rr0;
    g(2) = u(2,npts) - ur0;
    g(3) = u(3,npts) - er0;
  end
  iresout = ires;

function [flux, ires] = numflx(npde, t, x, ncode, v, uleft, uright, ires)

  global gamma;

  % Modified Harten-Lax-van Leer approximate Reimann solver.

  [flux, ifail] = d03pw( ...
                         uleft, uright, gamma);

function [u] = uvinit(x)

  global rl0 rr0 ul0 ur0 el0 er0;

  n = size(x,2);
  u = zeros(3,n);
  for i = 1:n
    if x(i)<1/2
      u(1,i) = rl0;
      u(2,i) = ul0;
      u(3,i) = el0;
    elseif x(i)== 1/2
      u(1,i) = (rl0+rr0)/2;
      u(2,i) = (ul0+ur0)/2;
      u(3,i) = (el0+er0)/2;
    else
      u(1,i) = rr0;
      u(2,i) = ur0;
      u(3,i) = er0;
    end
  end
```

9.2 Program Results

```
     d03pw example results

 Number of time steps           (nearest 50) =    800
 Number of function evaluations (nearest 50) =   1950
 Number of Jacobian evaluations (nearest  1) =      1
 Number of iterations           (nearest  1) =      2
```


Collision of two strong shocks, density
density
35
30
25
20
15
10
5
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
x
0
0.01
0.02
0.03
0.04
t


Collision of two strong shocks, velocity
velocity
20
15
10
5
0
-5
-10
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
t
1
0.8
0.6
0.4
0.2
0
x


Collision of two strong shocks, pressure
pressure
2000
1500
1000
500
0
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
x
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
t