

NAG Toolbox

nag_fit_pade_eval (e02rb)

1 Purpose

nag_fit_pade_eval (e02rb) evaluates a rational function at a user-supplied point, given the numerator and denominator coefficients.

2 Syntax

```
[ans, ifail] = nag_fit_pade_eval(a, b, x, 'ia', ia, 'ib', ib)
[ans, ifail] = e02rb(a, b, x, 'ia', ia, 'ib', ib)
```

3 Description

Given a real value x and the coefficients a_j , for $j = 0, 1, \dots, l$ and b_k , for $k = 0, 1, \dots, m$, nag_fit_pade_eval (e02rb) evaluates the rational function

$$\frac{\sum_{j=0}^l a_j x^j}{\sum_{k=0}^m b_k x^k}.$$

using nested multiplication (see Conte and de Boor (1965)).

A particular use of nag_fit_pade_eval (e02rb) is to compute values of the Padé approximants determined by nag_fit_pade_app (e02ra).

4 References

Conte S D and de Boor C (1965) *Elementary Numerical Analysis* McGraw–Hill

Peters G and Wilkinson J H (1971) Practical problems arising in the solution of polynomial equations *J. Inst. Maths. Applics.* **8** 16–35

5 Parameters

5.1 Compulsory Input Parameters

- 1: **a(ia)** – REAL (KIND=nag_wp) array
 $\mathbf{a}(j+1)$, for $j = 1, 2, \dots, l + 1$, must contain the value of the coefficient a_j in the numerator of the rational function.
- 2: **b(ib)** – REAL (KIND=nag_wp) array
 $\mathbf{b}(k+1)$, for $k = 1, 2, \dots, m + 1$, must contain the value of the coefficient b_k in the denominator of the rational function.
Constraint: if **ib** = 1, **b(1)** ≠ 0.0.
- 3: **x** – REAL (KIND=nag_wp)
The point x at which the rational function is to be evaluated.

5.2 Optional Input Parameters

1: **ia** – INTEGER

Default: the dimension of the array **a**.

The value of $l + 1$, where l is the degree of the numerator.

Constraint: $\mathbf{ia} \geq 1$.

2: **ib** – INTEGER

Default: the dimension of the array **b**.

The value of $m + 1$, where m is the degree of the denominator.

Constraint: $\mathbf{ib} \geq 1$.

5.3 Output Parameters

1: **ans** – REAL (KIND=nag_wp)

The result of evaluating the rational function at the given point x .

2: **ifail** – INTEGER

ifail = 0 unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

The rational function is being evaluated at or near a pole.

ifail = 2

On entry, $\mathbf{ia} < 1$,
or $\mathbf{ib} < 1$,
or $\mathbf{b}(1) = 0.0$ when $\mathbf{ib} = 1$ (so the denominator is identically zero).

ifail = -99

An unexpected error has been triggered by this routine. Please contact NAG.

ifail = -399

Your licence key may have expired or may not have been installed correctly.

ifail = -999

Dynamic memory allocation failed.

7 Accuracy

A running error analysis for polynomial evaluation by nested multiplication using the recurrence suggested by Kahan (see Peters and Wilkinson (1971)) is used to detect whether you are attempting to evaluate the approximant at or near a pole.

8 Further Comments

The time taken is approximately proportional to $l + m$.

9 Example

This example first calls nag_fit_pade_app (e02ra) to calculate the 4/4 Padé approximant to e^x , and then uses nag_fit_pade_eval (e02rb) to evaluate the approximant at $x = 0.1, 0.2, \dots, 1.0$.

9.1 Program Text

```
function e02rb_example

fprintf('e02rb example results\n\n');

ia = nag_int(5);
ib = nag_int(5);
ic = ia + ib - 1;
c = ones(ic,1);
for j = 3:ic
    c(j) = c(j-1)/double(j-1);
end

[a, b, ifail] = e02ra(ia, ib, c);

fprintf('      x          Pade        exp(x)        error\n');
for j=1:30
    x = j/10;
    [padx, ifail] = e02rb(a, b, x);
    f(j) = padx;
    expx = exp(x);
    relerr = abs(expx-padx)/expx;
    err(j) = relerr;
    if (mod(j,3)==1)
        fprintf('%6.1f%12.5f%12.5f%13.2e\n', x, padx, expx, relerr);
    end
end

x = [0.1:0.1:3];
fig1 = figure;
hold on;
plot(x,f,'s',x,log10(err),'o');
plot(x,exp(x),'Color','Magenta');
legend('Pade approximant','log_{10} error','exp(x)', 'Location','NorthWest');
title('The [4|4] Padé approximant of exp(x)');
xlabel('x');
hold off;
```

9.2 Program Results

e02rb example results

x	Pade	exp(x)	error
0.1	1.10517	1.10517	0.00e+00
0.4	1.49182	1.49182	1.04e-11
0.7	2.01375	2.01375	1.61e-09
1.0	2.71828	2.71828	4.05e-08
1.3	3.66930	3.66930	4.39e-07
1.6	4.95302	4.95303	2.91e-06
1.9	6.68580	6.68589	1.41e-05
2.2	9.02452	9.02501	5.47e-05
2.5	12.18030	12.18249	1.80e-04
2.8	16.43608	16.44465	5.21e-04

