NAG Toolbox

nag_matop_real_trapez_rq (f01qg)

1 Purpose

nag_matop_real_trapez_rq (f01qg) reduces the m by n ($m \leq n$) real upper trapezoidal matrix A to upper triangular form by means of orthogonal transformations.

2 Syntax

```
[a, zeta, ifail] = nag_matop_real_trapez_rq(a, 'm', m, 'n', n)
[a, zeta, ifail] = f01qg(a, 'm', m, 'n', n)
```

3 Description

The m by n ($m \leq n$) real upper trapezoidal matrix A given by

$$A = \begin{pmatrix} U & X \end{pmatrix},$$

where U is an m by m upper triangular matrix, is factorized as

$$A = \begin{pmatrix} R & 0 \end{pmatrix} P^{\mathrm{T}},$$

where P is an n by n orthogonal matrix and R is an m by m upper triangular matrix.

P is given as a sequence of Householder transformation matrices

$$P = P_m \cdots P_2 P_1,$$

the $(m-k+1)$th transformation matrix, P_k, being used to introduce zeros into the kth row of A. P_k has the form

$$P_k = \begin{pmatrix} I & 0 \\ 0 & T_k \end{pmatrix},$$

where

$$T_k = I - u_k u_k^{\mathrm{T}},$$

$$u_k = \begin{pmatrix} \zeta_k \\ 0 \\ z_k \end{pmatrix},$$

ζ_k is a scalar and z_k is an $(n-m)$ element vector. ζ_k and z_k are chosen to annihilate the elements of the kth row of X.

The vector u_k is returned in the kth element of the array **zeta** and in the kth row of **a**, such that ζ_k is in **zeta**(k) and the elements of z_k are in **a**$(k, m+1), \ldots,$ **a**(k, n). The elements of R are returned in the upper triangular part of **a**.

For further information on this factorization and its use see Section 6.5 of Golub and Van Loan (1996).

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Wilkinson J H (1965) *The Algebraic Eigenvalue Problem* Oxford University Press, Oxford

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(lda,:) – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least max$(1, \mathbf{m})$.

The second dimension of the array **a** must be at least max$(1, \mathbf{n})$.

The leading m by n upper trapezoidal part of the array **a** must contain the matrix to be factorized.

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the first dimension of the array **a**.

m, the number of rows of the matrix A.

When $\mathbf{m} = 0$ then an immediate return is effected.

Constraint: $\mathbf{m} \geq 0$.

2: **n** – INTEGER

Default: the second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq \mathbf{m}$.

5.3 Output Parameters

1: **a**(lda,:) – REAL (KIND=nag_wp) array

The first dimension of the array **a** will be max$(1, \mathbf{m})$.

The second dimension of the array **a** will be max$(1, \mathbf{n})$.

The m by m upper triangular part of **a** will contain the upper triangular matrix R, and the m by $(n - m)$ upper trapezoidal part of **a** will contain details of the factorization as described in Section 3.

2: **zeta**(**m**) – REAL (KIND=nag_wp) array

zeta(k) contains the scalar ζ_k for the $(m - k + 1)$th transformation. If $T_k = I$ then **zeta**$(k) = 0.0$, otherwise **zeta**(k) contains ζ_k as described in Section 3 and ζ_k is always in the range $\left(1.0, \sqrt{2.0}\right)$.

3: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= -1$

On entry, $\mathbf{m} < 0$,
or $\mathbf{n} < \mathbf{m}$,
or $lda < \mathbf{m}$.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The computed factors R and P satisfy the relation

$$(R0)P^{\mathrm{T}} = A + E,$$

where

$$\|E\| \leq c\epsilon\|A\|,$$

ϵ is the ***machine precision*** (see nag_machine_precision (x02aj)), c is a modest function of m and n and $\|.\|$ denotes the spectral (two) norm.

8 Further Comments

The approximate number of floating-point operations is given by $2 \times m^2(n - m)$.

9 Example

This example reduces the 3 by 5 matrix

$$A = \begin{pmatrix} 2.4 & 0.8 & -1.4 & 3.0 & -0.8 \\ 0.0 & 1.6 & 0.8 & 0.4 & -0.8 \\ 0.0 & 0.0 & 1.0 & 2.0 & 2.0 \end{pmatrix}$$

to upper triangular form.

9.1 Program Text

```
     function f01qg_example

fprintf('f01qg example results\n\n');

a = [2.4, 0.8, -1.4, 3,   -0.8;
     0,   1.6,  0.8, 0.4, -0.8;
     0,   0,    1,   2,    2];

[RQ, zeta, ifail] = f01qg(a);

disp('RQ Factorization of A');
disp('Vector zeta');
disp(zeta');
disp('Matrix A after factorization (R in left-hand upper triangle');
disp(RQ);
```

9.2 Program Results

```
     f01qg example results

RQ Factorization of A
Vector zeta
    1.2649    1.3416    1.1547

Matrix A after factorization (R in left-hand upper triangle
   -4.0000   -1.0000   -1.0000    0.6325   -0.0000
         0   -2.0000    0.0000    0.0000   -0.4472
         0         0   -3.0000    0.5774    0.5774
```