

NAG Toolbox

nag_matop_complex_trapez_rq (f01rg)

1 Purpose

nag_matop_complex_trapez_rq (f01rg) reduces the complex m by n ($m \leq n$) upper trapezoidal matrix A to upper triangular form by means of unitary transformations.

2 Syntax

```
[a, theta, ifail] = nag_matop_complex_trapez_rq(a, 'm', m, 'n', n)
[a, theta, ifail] = f01rg(a, 'm', m, 'n', n)
```

3 Description

The m by n ($m \leq n$) upper trapezoidal matrix A given by

$$A = (U \quad X),$$

where U is an m by m upper triangular matrix, is factorized as

$$A = (R \quad 0)P^H,$$

where P is an n by n unitary matrix and R is an m by m upper triangular matrix.

P is given as a sequence of Householder transformation matrices

$$P = P_m \cdots P_2 P_1,$$

the $(m - k + 1)$ th transformation matrix, P_k , being used to introduce zeros into the k th row of A . P_k has the form

$$P_k = \begin{pmatrix} I & 0 \\ 0 & T_k \end{pmatrix},$$

where

$$T_k = I - \gamma_k u_k u_k^H,$$

$$u_k = \begin{pmatrix} \zeta_k \\ 0 \\ z_k \\ cr \end{pmatrix},$$

γ_k is a scalar for which $\text{Re}(\gamma_k) = 1.0$, ζ_k is a real scalar and z_k is an $(n - m)$ element vector. γ_k , ζ_k and z_k are chosen to annihilate the elements of the k th row of X and to make the diagonal elements of R real.

The scalar γ_k and the vector u_k are returned in the k th element of the array **theta** and in the k th row of **a**, such that θ_k , given by

$$\theta_k = (\zeta_k, \text{Im}(\gamma_k)),$$

is in **theta**(k) and the elements of z_k are in **a**($k, m + 1$), ..., **a**(k, n). The elements of R are returned in the upper triangular part of **a**.

For further information on this factorization and its use see Section 6.5 of Golub and Van Loan (1996).

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Wilkinson J H (1965) *The Algebraic Eigenvalue Problem* Oxford University Press, Oxford

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(*lda*, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least max(1, **m**).

The second dimension of the array **a** must be at least max(1, **n**).

The leading *m* by *n* upper trapezoidal part of the array **a** must contain the matrix to be factorized.

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the first dimension of the array **a**.

m, the number of rows of the matrix *A*.

When **m** = 0 then an immediate return is effected.

Constraint: **m** ≥ 0 .

2: **n** – INTEGER

Default: the second dimension of the array **a**.

n, the number of columns of the matrix *A*.

Constraint: **n** $\geq m$.

5.3 Output Parameters

1: **a**(*lda*, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** will be max(1, **m**).

The second dimension of the array **a** will be max(1, **n**).

The *m* by *m* upper triangular part of **a** will contain the upper triangular matrix *R*, and the *m* by (*n* − *m*) upper trapezoidal part of **a** will contain details of the factorization as described in Section 3.

2: **theta**(**m**) – COMPLEX (KIND=nag_wp) array

theta(*k*) contains the scalar θ_k for the (*m* − *k* + 1)th transformation. If $T_k = I$ then **theta**(*k*) = 0.0; if

$$T_k = \begin{pmatrix} \alpha & 0 \\ 0 & I \end{pmatrix}, \quad \operatorname{Re}(\alpha) < 0.0$$

then **theta**(*k*) = α , otherwise **theta**(*k*) contains θ_k as described in Section 3 and $\operatorname{Re}(\theta_k)$ is always in the range (1.0, $\sqrt{2.0}$).

3: **ifail** – INTEGER

ifail = 0 unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = -1

On entry, $\mathbf{m} < 0$,
or $\mathbf{n} < \mathbf{m}$,
or $lda < \mathbf{m}$.

ifail = -99

An unexpected error has been triggered by this routine. Please contact NAG.

ifail = -399

Your licence key may have expired or may not have been installed correctly.

ifail = -999

Dynamic memory allocation failed.

7 Accuracy

The computed factors R and P satisfy the relation

$$(R \ 0)P^H = A + E,$$

where

$$\|E\| \leq c\epsilon\|A\|,$$

ϵ is the **machine precision** (see nag_machine_precision (x02aj)), c is a modest function of m and n , and $\|\cdot\|$ denotes the spectral (two) norm.

8 Further Comments

The approximate number of floating-point operations is given by $8 \times m^2(n - m)$.

9 Example

This example reduces the 3 by 4 matrix

$$\begin{pmatrix} 2.4 & 0.8 + 0.8i & -1.4 + 0.6i & 3.0 - 1.0i \\ 0 & 1.6 & 0.8 + 0.3i & 0.4 + 0.5i \\ 0 & 0 & 1.0 & 2.0 - 1.0i \end{pmatrix}$$

to upper triangular form.

9.1 Program Text

```
function f01rg_example

fprintf('f01rg example results\n\n');

a = [ 2.4,      0.8 + 0.8i, -1.4 + 0.6i,  3    - i;
      0 + 0i,    1.6 + 0i,   0.8 + 0.3i,  0.4 + 0.5i;
      0 + 0i,    0    + 0i,   1    + 0i,   2    - i];
[RQ, theta, ifail] = f01rg(a);
```

```
disp('RQ Factorization of A');
disp('Vector theta');
disp(theta');
disp('Matrix A after factorization (R in left-hand upper triangle');
disp(RQ);
```

9.2 Program Results

f01rg example results

```
RQ Factorization of A
Vector theta
  1.2924 + 0.0000i   1.3861 + 0.0000i   1.1867 + 0.0000i

Matrix A after factorization (R in left-hand upper triangle
 -3.5808 + 0.0000i   0.2533 - 0.9059i   -2.2862 - 0.6532i   0.5120 + 0.2601i
  0.0000 + 0.0000i  -1.7369 + 0.0000i   -0.4491 - 0.6940i  -0.2544 - 0.1187i
  0.0000 + 0.0000i   0.0000 + 0.0000i   -2.4495 + 0.0000i   0.6880 + 0.3440i
```
